BC 1 Rate of Change Activity Sheet Teacher Notes

Preliminaries:

Before beginning this section of handouts, students will be introduced to a variety of vocabulary words often associated with calculus. These words will be used in an intuitive sense only and will not have been formally defined. Vocabulary should include graphical terms such as continuous, increasing, decreasing, maximum and minimum points, concave up, concave down, and point of inflection. In addition, discussion of the concept of "rate of change" should begin. It should be mentioned that many quantities change – population, cost, and temperature, to name just a few. All that is specifically required at this point can be related simply to physical motion. Students are expected to have some sense of velocity. For example, if velocity is higher, how does that affect the object's position?

Goals:

These activity sheets attempt to develop a student's intuition for the concept of a rate of change, particularly as a function in its own right. As stated above, this requires only a general sense of motion. In working together, students' vocabulary and clarification of notions of velocity will improve. No sense of mastery is desired or expected. The sheets are meant to be done informally. Students will see how zero and constant velocities affect position. In addition, they will begin to relate positive and negative velocities with increasing and decreasing position functions. More generally, students will begin to see that one function can have an associated function that represents the first function's rate of change. In other words, we are beginning to create the derivative function. From these sheets, it will be easy to develop very simple cases of Euler's method.

Activity Sheet 1:

This sheet should go fairly quickly, and that is intended. While some students will ask if there is more to this, others will benefit from the discussions, which may not be all that obvious to all. It is important to be sure your students do not skip over the ideas.

Problems (1) and (2) suggest a couple of points. First, if s is constant, then $v = 0$. Second, the value of the constant for s is immaterial.

Problems (3) and (4) reinforce the concept that the initial position does not affect the velocity. Also, if position is linear, velocity is constant. Some students will likely suggest this, but be careful to question this. How do you know? Are you sure? Can we explain this? At this point, one can probably make the idea seem reasonable with an example or two about a car.

Problems (5) and (6) suggest that the steeper the linear position graph, the greater the velocity.

Problems (7) and (8) introduce the idea of negative velocity. In addition, these problems illustrate that when there is a linear position graph with negative slope, steeper means lesser velocity, since velocity is negative, though greater *speed* (magnitude of velocity).

For problems (11) and (12), ignore issues of endpoints and connections. Students will probably be happy to ignore the issue.

Activity Sheet 2:

The descriptions here are most important. Try to get your students to clarify their understanding of the changes of the position of the object, not simply the velocity. Many of these problems are considerably more difficult and do not have clear or obvious answers. In particular, problems (1), (2), or (3) could have a linear, exponential or other power function velocity. This cannot be determined from the information given (due to a lack of scale on each axis). The discussion among the students is the important part at this time. If students' graphs seem less than wonderful, that's quite acceptable at this point, though major misconceptions should be addressed. Again, the goal is to get students to think and to talk. Don't wait for everyone to be good at these graphs before continuing.

In my class, I ask students to put lots of graphs on the board and then discuss what they have in common, what looks good, what looks bad, etc. This generates great discussions!

Activity Sheet 3:

This sheet reverses the process. Begin with the velocity and determine as much as possible about the position function. Students may start with $s(0) = 0$, but this is not necessary, though it should be pointed out, if all students appear to be using $s(0) = 0$, that $s(0)$ can have any value. The natural instinct of most students will be to connect the pieces of their graphs. Suggest this if a student asks. It is good to mention at some point that velocity normally cannot change in this way, but that velocity is continuous under usual circumstances. These graphs are very rough approximations to the real velocity. (This is being done as a lead-in to Euler's method, so you might tell your students that there is a reason for this!) State that it is simply much easier to deal with constant velocities, so that's what many of these do. The graph of problems (3) and (4) show open circles to indicate no endpoints. This is to be assumed in problems from this point on, but this won't be an issue for most students. If asked, simply state that these are approximations to a real velocity graph, so specific points should not be of concern.

Activity Sheet 4:

Here, we are more specific about joining pieces of the position function and about starting at $y(0) = 0$. Also, remind your students again that many phenomena change and that velocity and position are not always appropriate words. We use y for the function and y' for its rate of change. Care should be taken here to clarify relative rates. How does a larger rate of change affect the function? In problem (3), the rates are about the same but in opposite directions. Since the negative rate is for a longer period of time, where will the function end? Problem (4) should give two pairs of parallel line segments. And in (6), if you think "trig," what is y'? And what is y? Hmmm...

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