

Monotonic sequences are sequences that are (essentially) increasing or decreasing. That is, either $a_1 \leq a_2 \leq a_3 \leq \dots$ or $a_1 \geq a_2 \geq a_3 \geq \dots$. Sometimes, we make the distinction between monotonic and strictly monotonic sequences. In a strictly monotonic sequence, equality between terms is not allowed. That is, $a_1 < a_2 < a_3 < \dots$ or $a_1 > a_2 > a_3 > \dots$

A sequence is bounded if there are real numbers K and M such that $K \leq a_n \leq M$ for all n .
(HH P.466)

- (1) Which of the sequences below is monotonic? (List the letters to indicate the sequence.)

- (2) Which of the sequences below is bounded?

- (3) Which of the sequences below is convergent?

$$a_n = \frac{1}{n^2}$$

$$b_n = \sin(n)$$

$$c_n = \frac{(-1)^{n+1}}{n}$$

$$d_n = 2^n$$

$$e_n = (-2)^n$$

$$f_n = (0.9)^n$$

$$g_n = (-0.9)^n$$

$$h_n = \frac{3^n}{n^3}$$

$$j_n = \frac{n^3}{3^n}$$

$$m_n = \left(1 + \frac{1}{3n}\right)^n$$

$$p_n = \frac{3 + \sin(n)}{n}$$

$$q_n = \frac{\cos(n\rho)}{n^2}$$

$$r_n = \frac{2n^3 + 1}{5n^3 + 3n^2 + 2}$$

$$s_n = \arctan n$$

$$t_n = \sqrt[n]{4n}$$

(4) Give an example of a sequence for each of the conditions listed below, if possible.

(a) monotonic, convergent

(b) not monotonic, convergent

(c) not bounded, monotonic, convergent

(d) bounded, not monotonic, divergent

(e) not bounded, divergent