Population...logistically speaking...

(1) Basic population growth says that the rate of change of the population $P$ is proportional to the population itself. Express this as a differential equation and state the general solution.

(2) What’s the problem with this model?

(3) This problem leads us to another model, called the logistic model of population growth. It states that the rate of growth of the population is proportional both to the population itself and to the difference between the carrying capacity and the population. State this as a differential equation, using $C$ as the carrying capacity and $P$ for the population.

(4) Let’s consider a simple case: $P' = .15P(2 - P)$ Here, the carrying capacity is __________, which may represent 200 or 2000, for example. (The value 1 is often used to represent 100%.) The slope field is shown below. On it, sketch two possible solution curves, one with $P(0) = 0.1$ and one with $P(0) = 0.5$. 

![Slope field image]
(5) Again with $P' = .15P(2 - P)$ and beginning with $P(0) = 0.1$, do 3 steps of Euler’s method "by hand" with $\Delta t = 1$ to approximate $P(3)$. (Make a chart or show the equations. Check your work with your calculator.)

(6) Solve the differential equation $P' = .15P(2 - P)$ to find a general solution by separating variables. (Do not use the table of integrals! After doing the integration, hints for the algebra to solve for $P$ are on P. 589.)
(7) Again, given \( P(0) = 0.1 \), find the specific solution. (Graph this on your calculator to see if it looks right.)

(8) Evaluate \( P(3) \) using your solution function. Compare this to the Euler’s method approximation from problem (5).

(9) Consider the graph of \( P' = 0.15P(2 - P) \) as \( f(x) = 0.15x(2 - x) \). For what value of \( x = P \) does \( f = P' \) have a maximum? What does this state about the population growth in this case? What does this state in general for the DE \( P' = kP(C - P) \)?

(10) What would have happened if the initial population had been 2.5? Explain.

Note: Some texts (including HH) give another form of the logistic DE: \( \frac{dP}{dt} = kP \left( 1 - \frac{P}{C} \right) \). This is equivalent to the form shown here with \( C \) factored out. Thus, the values of \( k \) are different in the two models. You should recognize both models as representing logistic growth.