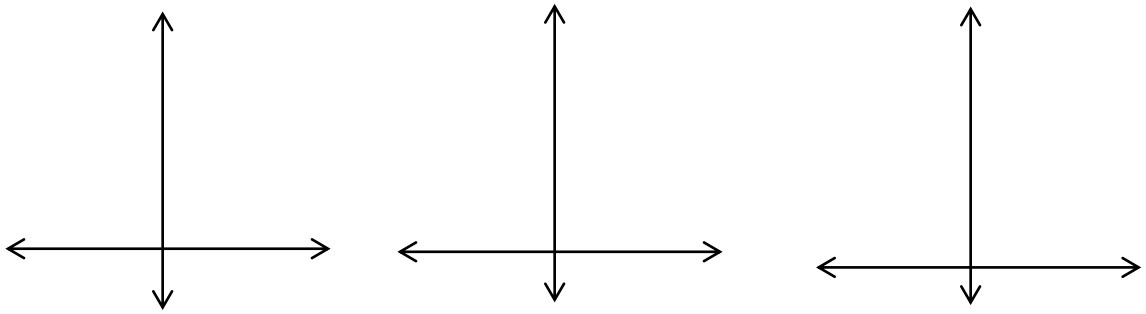
Derivatives of Exponential Functions Name:_____

This activity sheet is designed to find the derivative of the function $f(x) = b^x$.

(1) Graph each function (——) and its derivative (- - -). Try the window [–2, 4] x [–2, 20].

$f(x) = 2^x$ $g(x) = 3^x$ $h(x) = e^x$



(2) What can be said about h and h'?

(3) To investigate the derivative of $f(x) = b^x$, we consider the ratio of the derivative to the function itself. In other words, we want to consider the set of ordered pairs of the form (b, ratio of y'/y) for a variety of values of b.

Begin with the specific case where $b = 2$. On the "y=" screen, let :

(TI-89) y1=2^x, y2 = nDeriv(y1(x), x), and let y3 = y2(x)/y1(x) OR

(TI-84) Y_1 = 2^X, Y_2 = nDeriv(Y_1, X, X) and $Y_3 = Y_2/Y_1$

Use "Table." What is true about $y3$? What is its value?

Change $y1$ and repeat this process for each of the values of b given below. Fill in the ordered pairs with $y3$ = ratio of y'/y. Plot these ordered pairs on the graph.

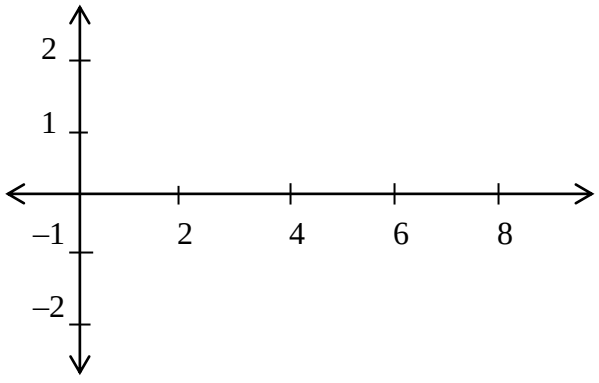
(2, _____), (.25, _____)

(.5, _____), (1.5, _____)

(3, _____), (4, _____)

(5, _____), (6, _____)

(7, _____), (8, _____)



What does this function look like?

In short, we have the relationship (*b*, _____). This means that the ratio

of $\frac{\frac{d}{dx}(b^x)}{b^x}$ = _____ , so the derivative of $f(x) = b^x$ is $\frac{d}{dx}\left(b^x\right)$ = _____ .

(4) We've seen that the derivative of $y = b^x$ is a multiple of the function itself. Show this by setting up the derivative by definition and factoring out b^x. The remaining expression may be seen as the derivative of _____ at a = _____.

(5) What does your derivative rule imply about the derivative of $y = e^x$? Show.

(6) Find the derivative of each of the following using the rule found above. (For some, you will want to rewrite the expression using properties of exponents.)

$y = 10^x$ $y = 4 \cdot 3^x$ $y = 4^{2x}$

$y = (1/2)^x$ $y = \frac{3}{5^x}$

$y = x^2 + 2^x + 2$ $y = 3 \cdot 6^{-x}$

$y = e^{2x} + x^{2e}$ $y = 3e^{x+2} + 5^{x-1}$