Nice and Not-so-Nice Limits

Nice Limits

Some limits seem obvious. Recall the graph of $k(x)=\frac{1}{7}\left(x^3-x^2-7x+7\right)$ on "Limits 1". Here, $\lim\limits_{x\to 0}\left(\frac{1}{7}\left(x^3-x^2-7x+7\right)\right)=1$, as we would all guess. This is because the function $k(x)=\frac{1}{7}\left(x^3-x^2-7x+7\right)$ is a well-behaved function near $x = 0$. In other words, as x gets closer to 0, $k(x)$ gets closer to 1, so we say the limit equals 1. The behavior of the function is predictable.

As long as the function is nice (continuous in an open interval containing $x = a$), we may evaluate the limit by simply evaluating the function at $x = a$.

(1) $\lim\limits_{x\to 2}(5x-1)=$ (2) $\lim\limits_{x\to 3}|x-1|=$ (3) $\lim\limits_{x\to \frac{\pi}{6}}\frac{\cos x}{\sin(3x)}=$

Not-so-Nice Limits

Unfortunately, lots functions are not continuous at all points; thus, the limits at these points may not be quite so obvious.

Example: $\lim\limits_{x\to 2}\frac{x^2+x-6}{2x-4}$. The function here is $f(x)=\frac{x^2+x-6}{2x-4}$ and the point in question is $x = 2$. Look at this problem numerically first. Use the table on your calculator to fill in the following chart. Under **TBLSET**, you will need to change the **Independent** setting from **AUTO** to **ASK** in order to enter these x values directly into the table.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				undef			

What do the values in the table above suggest?

Now sketch the graph of f.
(Use ZoomDec with xres = 1 on the TI-89.)
(Use ZDecimal with xres = 1 on the TI-84.)

Describe and explain your graph.
Hint:
Recall what you learned about rational functions.

Example continued: Now, it's time to do some algebra. Factor, simplify the expression and complete the following:

$$\lim_{x\to 2}\frac{x^2+x-6}{2x-4}=\lim_{x\to 2}$$

What happens in the graph at $x = 2$?
Does your analytical work confirm your graphical and numerical work?

Many of these not-so-nice limits may be calculated by doing some algebra first, simplifying, and then evaluating the resulting expression. Do the following limits analytically. Confirm your work by using tables and/or graphics.

(4) $\displaystyle\lim_{x\to -1}\frac{x^2-3x-4}{x^2+7x+6}$

(5) $\displaystyle\lim_{h\to 0}\frac{3(x+h)^2-3x^2}{h}$

(6) $\displaystyle\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}$

(7) $\displaystyle\lim_{x\to 5}\frac{\frac{2}{x}-\frac{2}{5}}{x-5}$

The following limits cannot be determined by simple algebraic methods. Use graphs and tables to estimate/guess each limit.

(8) $\displaystyle\lim_{x\to 0}\frac{\sin(3x)}{2x}$ (9) $\displaystyle\lim_{x\to 1}\frac{\ln x}{x-1}$ (10) $\displaystyle\lim_{x\to 2}\frac{3^x-9}{x-2}$