BC 3 Name:________________________________

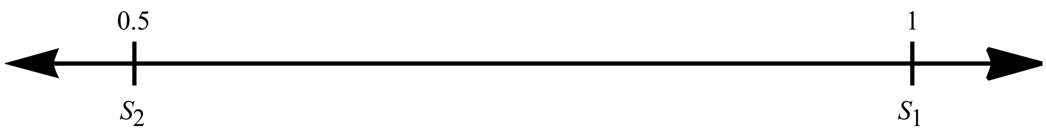
Alternating Series

Defn: A series $\sum a_k$ is an alternating series if, with $a_n > 0$, it has the form

$$a_1 - a_2 + a_3 - a_4 + \dots \quad \text{OR} \quad -a_1 + a_2 - a_3 + a_4 - \dots$$

(1) Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$, the alternating harmonic series.

Find $S_1, S_2, S_3, \dots, S_{10}$ to two decimal places, and plot each of these on the line below. (This has been started for you.)



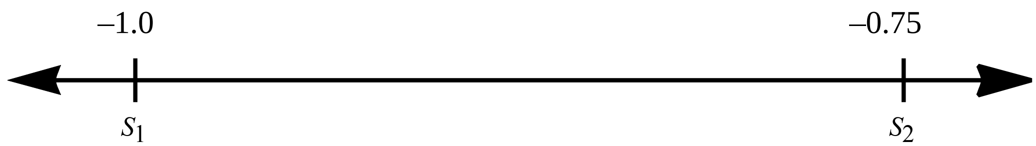
Describe the pattern of the S_n for even and odd values of n.

Based on this pattern, do you think the series converges?

Where is S? (Write an inequality involving S using numbers you calculated above.)

(2) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -1 + \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \cdots$

Find $S_1, S_2, S_3, \ldots, S_{10}$ to three decimal places, and plot each on the line below. (Do not try to plot these precisely to scale!)



Describe the pattern of the S_n.

How does this differ from the pattern in the first example?

Do you think this series converges?

Where is S? (Write an inequality involving S.)

It is true that both of the preceding series converge since the terms of both followed certain patterns. For a series $\sum a_k$, what are these conditions that will guarantee convergence? (OK, the first condition is given for you. But what else must be true about the a_n?)

(i) The series is alternating.

(ii)