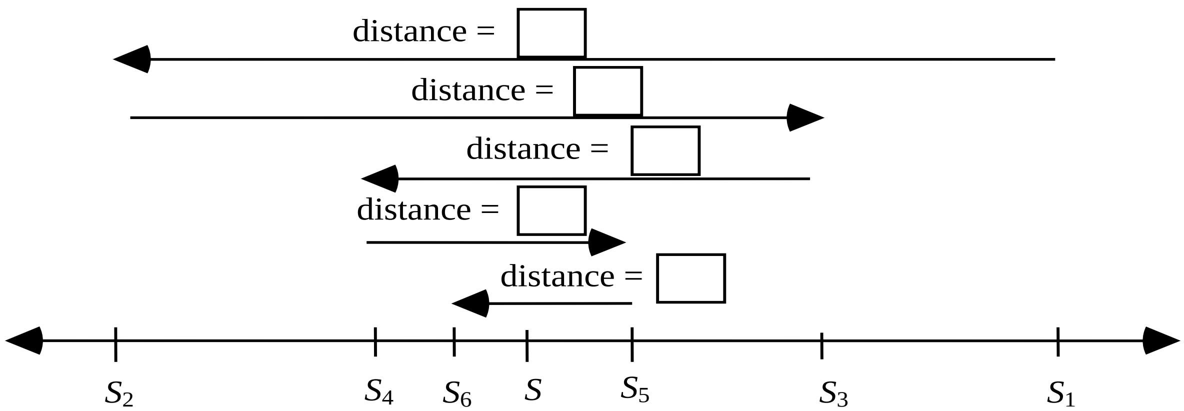
BC 3 Name: ______________________________

Approximating Alternating Series

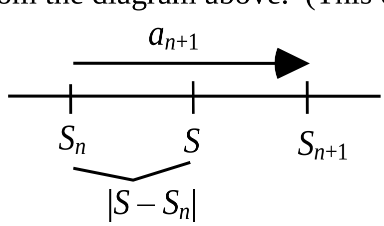
Consider the convergent, alternating series $\sum_{k=1}^{\infty} a_k = a_1 + a_2 + a_3 + \cdots$ That is, the alternating signs are "built in" to the a_k terms, and we will also assume here that $| a_k |$ decreases to 0. Fill in the (positive) distances, in terms of an a_i, given by each arrow.



Usually, it is impossible for us to calculate the value of S directly. Therefore, we will approximate the infinite sum S by using S_n for some value of n. For example, let's use S_5 to approximate S. The next real question, or problem, is to find out just how good S_5 is as an approximation for S. In other words, we need to know how close S_5 is to S. We can express this distance as $\left| S - S_5 \right|$. Usually, we cannot calculate this distance exactly. Why not? (This may or may not be obvious.)

From the figure above, we can see that $\left| S - S_5 \right| < \left| S_6 - S_5 \right| = \left| a_6 \right|$. Thus, we know that the error made by using S_5 as an approximation for S is less than the value of $|a_6|$. More generally, we need to look at the size of $\left| S - S_n \right|$. Since the S_n bounce back and forth around S, in smaller and smaller steps, then S is between S_n *and* S_{n+1} for all n.

Consider an arbitrary stage from the diagram above. (This example has $a_{n+1} > 0$.)



Here, we can see that $| S - S_n | <$ _______. More specifically, we have

$| S - S_3 | <$ _______, $| S - S_5 | <$ ________, and $| S - S_{20} | <$ ________.

(1) Consider the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

(a) If we approximate S by using the first 10 terms of this series, what will be the magnitude of the error? In other words, how large is $|S - S_{10}|$?

(b) Find S_{10} and write an inequality (without absolute values) that bounds S.

(c) How many terms of the series need to be used to be sure that the error made when using the partial sum formed by these terms to approximate S is less than 0.01?

(2) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$.

(a) If we approximate S by using the first 15 terms of this series, what will be the magnitude of the error? In other words, how large is $|S - S_{15}|$?

(b) Find S_{15} and write an inequality (without absolute values) that bounds S.

(c) How many terms of the series need to be used to be sure that the error made when using the partial sum formed by these terms to approximate S is less than 0.01?

(3) Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n}}{n^3}$.

(a) If we approximate S by using the first 19 terms of this series, what will be the magnitude of the error? In other words, how large is $|S - S_{19}|$?

(b) How many terms of the series need to be used to be sure that the error made when using the partial sum formed by these terms to approximate S is less than 0.01?