Name:______________________________

(1) Let $f(x) = \ln(1 + x)$ and let $g(x) = C_0 + C_1x + C_2x^2 + C_3x^3$. Set $f(0) = g(0)$, and set the first three derivatives evaluated at $x = 0$ equal to each other in order to find the values of the C_i and the polynomial $g(x)$. (In other words, set $f^{(k)}(0) = g^{(k)}(0)$ for $k = 0, 1, 2, 3$.)

(2) Find both $f(a)$ and $g(a)$ for each of the following values of a. Are they very close to each other? When are the values closer?

	$f(a)$	$g(a)$
$a = 1.0$		
$a = 0.5$		
$a = 0.1$		

(3) Use your calculator to graph both functions f and g. Try different windows to get a good view and copy the graph below. On what interval does g seem to be fairly good as an approximation of f?

(4) Now sketch a graph of $y = |\text{ error }| = |\, f(x) - g(x) \,|$. This represents the absolute value of the error, or the distance between the function f and its approximation g. Describe the graph and what this says about g as an approximation.

(5) Extend g by continuing the pattern of the coefficients of g to form an infinite series that will approximate the function f. (Take whatever pattern is most obvious, even if three terms hardly offer a guarantee.) Use an appropriate test to determine the open interval for which this series converges.

Check the endpoints of your interval to determine the interval of convergence.

(6) If we use only the three terms of our polynomial g – the first three terms of our infinite series – and we use $x = 0.75$, find an upper bound for the error. Find S_3 and use this to write an inequality for S, the infinite sum.

(7) Again using these three terms and assuming $x > 0$, what are the possibilities for x if the error is to be less than 0.005? Why is it important to consider $x > 0$ only at this point?

(8) If $0 < x < 0.5$, how many terms of the series do you have to use in order to get an error which is less than 0.001? (Show the set-up clearly.)