

- (1) Let $f(x) = \tan^{-1}x$ and let $g(x) = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4$. Set $f(0) = g(0)$, and set the first four derivatives evaluated at $x = 0$ equal to each other in order to find the values of the C_i and the polynomial $g(x)$. (In other words, set $f^{(k)}(0) = g^{(k)}(0)$ for $k = 0, 1, 2, 3, 4$.) (Be careful with the derivatives of f . They aren't real friendly.)

- (2) Find both $f(a)$ and $g(a)$ for the following values of a . What do you notice?

$f(a)$

$g(a)$

$a = 2$

$a = 1$

$a = 0.5$

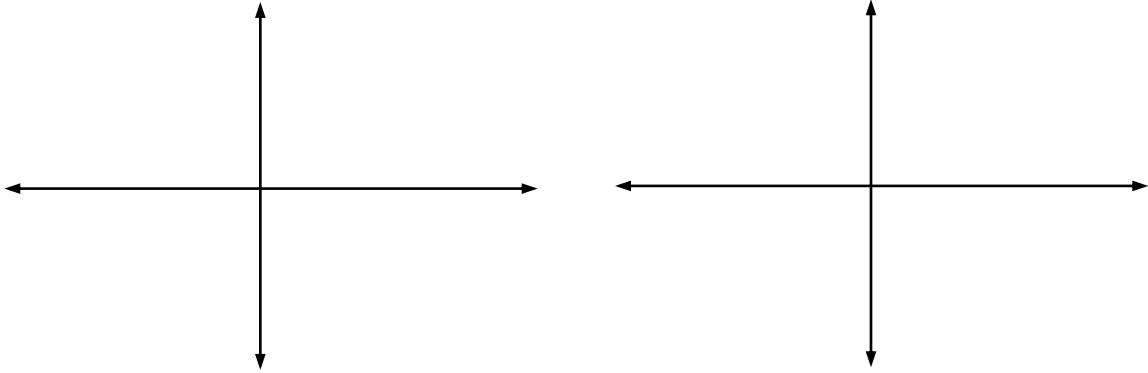
$a = 0.1$

What will happen for negative values of a ?

- (3) Sketch the graphs indicated below. Try different windows to find an appropriate view. On what interval does g seem to be fairly good as an approximation of f ?

Sketch both f and g .

Sketch $y = |\text{error}| = |f(x) - g(x)|$.



- (4) Continue the pattern, extending g to form an infinite series that will approximate the function f . (Yes, you only have two terms, so a pattern is hardly clear. Hints: The terms do alternate, and the pattern is nice and simple.) Use an appropriate test to determine the open interval for which this series converges.

Check the endpoints and state the interval of convergence.

(5) If we use only two terms of g and we use $x = 0.75$, find an upper bound for the error. Use this to find an interval for S .

(6) Again using these two terms, what values of x may be used if the error will be less than 0.005?

(7) If $|x| < 0.5$, how many terms of the series must be used to ensure that the error will be less than 0.001?