BC 3 Name:__________________________________
Series 6

New from Old
Complete the right side of the equations below with the appropriate series and the interval of convergence if requested.

(1) $\sin x =$

Differentiate both sides.

Does this "work"?

Differentiate both sides again. Does this still "work"?

(2) $e^x =$

Integrate both sides.

Does this "work"? What must you <u>always</u> include when integrating?

(3) Find the series for the first function below. Show four terms and state the interval of convergence.

<u>Interval of convergence</u>

$$\frac{1}{1-x} =$$

For each of the next two functions, determine whether substitution, differentiation, or integration of the function above will give the next function. Then do this to the terms on the right to find 4 terms of the series. State the interval of convergence.

$$\frac{1}{1+x} =$$

$\ln(1 + x) =$

Note the endpoints, particularly on the integral.

(4) Find each series. (Same as for problem 3.) Show four terms.

Interval of convergence

$\frac{1}{1-x} =$

$\frac{1}{1+x^2} =$

$\tan^{-1}x =$

Note that this is the second method we've seen for finding the Maclaurin series for $\tan^{-1}x$. Any preference?

Theorem: (Simplified) Taylor series may be differentiated or integrated term by term and will represent the appropriate function on the same open interval.

Note: Convergence or divergence may change at each endpoint.

(5) Find, using "simple" multiplication.

$x^2e^x =$

$4x\cos(2x) =$

(6) Multiplication of series. Find the series for $\frac{e^x}{1-x}$ by multiplying enough terms of each of the two series together. (Be careful. Think ahead.)

(7) Let $f(x) = \dfrac{5x - 1}{x^2 - x - 2}$. Finding the derivatives of this to create the series directly would be somewhat unpleasant. Try this approach:

(a) Use partial fractions to decompose f.

(b) Write the series for each of the two partial fractions. (One may require a bit of sneakiness.)

(c) Add and simplify.

(8) Let's try that last function by an additional method: Long division. Find three terms.

$$-2 - x + x^2 \overline{\big) -1 + 5x}$$