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Series 07

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Series 7 – Lagrange Error

The Lagrange error bound for Taylor series states the following:

$$|\text{error}| \leq \frac{K_{n+1} \times |x - a|^{n+1}}{(n+1)!}, \text{ where } |f^{(n+1)}(x)| \leq K_{n+1}$$

So where does this come from? Here are some clues.

Here, let $n = 2$ and let $a = 0$. Assume $x > 0$. (The other case is similar.) By our notation, we'll assume $|f^{(3)}(x)| \leq K_3$, so that

$$-K_3 \leq f^{(3)}(t) \leq K_3$$

Take the integral of each part of the inequality from $0 \rightarrow x$.

Replace all of your x 's with t 's. (It really is better math!)

Once again, integrate each term in your inequality from $0 \rightarrow x$.

Replace all of your x 's with t 's. (It's still better math.)

Once again, integrate each term in your inequality from $0 \rightarrow x$.

In the middle of your three-part inequality, if you made it this far, you should have:

$$f(x) - f(0) - f'(0) \times x - f''(0) \times \frac{x^2}{2}. \text{ This is } f(x) - P_2(x).$$

Now, use absolute value to rewrite the three-part inequality:

$$|f(x) - P_2(x)| \leq \underline{\hspace{2cm}} \quad (\text{Yea!})$$

More on the Lagrange error bound

A slightly different form: $|\text{error}| = |R_n| = |f(x) - P_n(x)| = \left| \frac{f^{(n+1)}(c)}{(n+1)!} \times (x-a)^{n+1} \right|$ for some c between a and x . Given a value of n and a value of x , the remaining issue is to find an upper bound for the value of $|f^{(n+1)}(c)|$, given as K_{n+1} . In some cases, one may take into account restrictions on the value of c and then work to find an upper bound. In other cases, we simply look at the size of the $(n+1)^{\text{st}}$ derivative for all values of x , sometimes referred to as K_{n+1} .

- (1) A function f has derivatives of all orders at $x = 0$ such that $|f^{(n)}(x)| \leq \frac{n}{4}$ for all x . Find an upper bound for the size of the error when using the Maclaurin polynomial with $x = 1.2$ and $n = 6$.

- (2) A function f has derivatives of all orders at $x = 2$ such that $|f^{(n)}(x)| \leq n^2$ for all x . If $x = 1.2$, find an upper bound for the size of the error when the function f is approximated by the 4th degree Taylor polynomial at $x = 2$.

- (3) Look back at worksheet Series 1 where we found the Maclaurin series for $f(x) = \ln(1+x)$. At that time, we used values of $x > 0$ so that the series was alternating. For values of $x < 0$, the series no longer alternates, and we must look at the Lagrange bound. If $x = -0.3$ and $n = 3$, find an upper bound for the size of the fourth derivative by graphing this function over the interval $[-0.3, 0]$. Then use this maximum size to find an upper bound for the error.