Using the Lagrange error bound on the Maclaurin series for e^x
(Assume we know $e < 3$, $e^2 < 9$, etc.)

(1) If P_8, the 8th degree Maclaurin polynomial (with terms through $n = 8$) is used to approximate e^2, find an upper bound for the error.

(2) Approximate e with an error less than 0.0001. How many terms are necessary?

(3) If terms through $n = 6$ are used to approximate $e^{3/2}$, find the error.

(4) To approximate $e^{9/10}$ with an error less than 0.005, how many terms are necessary?

(5) To use $1 + x + \frac{x^2}{2}$ to approximate e^x with an error less than 0.05, and we know $|x| < 1$, what values of x can be used? (Careful! Two cases here.)

Lagrange error bound on the Maclaurin series for $\cos(x)$ and $\sin(x)$.

(6) If $|x| < 0.3$ and we use $1 - \frac{x^2}{2}$ to approximate $\cos(x)$, find the error

(a) by using the Lagrange bound on the second degree polynomial.

(b) by using the Lagrange bound by considering the third degree polynomial $1 - \frac{x^2}{2} + 0x^3$.

(c) Compare these bounds. Is it mathematically correct to use either one? Why? Which is better? One of these is the same as the Alternating Series error approximation. Which one? Why?

(7) If 3 (non-zero) terms of the series for $\cos x$ are used to approximate $\cos(1.8)$, find an upper bound for |error|.

(8) If 4 (non-zero) terms of the series for $\sin(x)$ are used to approximate $\sin(x)$ and the size of the error is to be less than 0.05, what values of x may be used?

(9) To approximate $\cos(3)$ with an error less than 0.005, how many terms must be used?