Build Student Engagement, Mental Math, and Reasoning with “Math Talks”

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Let’s try a math talk

- Facilitator presents prompt
- Participants generate multiple methods
- Signal number of methods with fingers
- Participants share methods
- Facilitator scribes
- Discussion
Math Talk: Number of the Day

- Think of as many different ways as you can to express the number 36.
- When you have one way, put your thumb up in front of your chest.
- When you think of more ways, put out fingers too.
Discuss with shoulder partner

What stood out to you about this math talk?

Questions we may address today:

• What are some of their benefits and challenges?
• What would it take to do a math talk in your class?
• How can you and colleagues use math talks in your school and district?
Defining Math Talks

Math Talks are short daily rituals with the entire class to develop conceptual understanding of and efficiency with numbers, operations and mathematics, through student discussion and engagement.

Math Talks are used to:

• Review, practice, or reinforce procedures and concepts
• Introduce concepts and properties concretely
• Explore mathematical connections and relationships
Some Possible Responses

\[ 18 + 18 \quad 25.65 + 10.35 \]
\[ 6^2 \quad 9 \div 1/4 \]
\[ 2^2 \cdot 3^2 \quad -15 + 51 \]
\[ 9+9+9+9 \quad 3\sqrt{144} \]
Possible Constraints

• Number of operations (exactly, at least, at most)
• Exponents/roots
• Distributive properties
• Field properties
• Expressions or equations with variables
• Inequalities
• Consecutive numbers
• Integers
• Rational numbers
• Set of clue statements to identify the number
Model of a Math Talk

The number for today is 15.

Write as many expressions as you can that are equal to 15. All the numbers in each expression must be even. Please use two or more operations in each expression.
Determine equivalent expressions/representations
Dot Patterns:

How many dots?

How did you see it?
Sharing
Getting Started: Dot Talks

- Often “takes answer off the table”
- Provides access to all students
- Promotes student confidence
- Develops math vocabulary
- Allows for multiple strategies
- Highly amenable to connections
Math Talk: Number Line

Place the numbers on the number line

- $\frac{5}{8}$
- 0.75%
- 0.45
- 67%
- $\frac{3}{6}$
- $\frac{2}{3}$
- $\frac{3}{4}$
- $\frac{5}{6}$
- 62.5%
- 0.65
Math Talk: Number Line

*Processing with the Class*

- Student use paper to create a number line.
- They place the number in the appropriate location on the line.
- Call on students to come forward and place the number on the line (post it notes).
- After all numbers are placed, ask class if anyone wants to re-arrange to be more accurate.
Math Talk: Which Two Are Closer?

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<table>
<thead>
<tr>
<th></th>
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<tr>
<td>1/4</td>
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<td>0.03</td>
<td>0.16</td>
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<td>1 7/8</td>
<td>2 1/5</td>
<td>1 3/8</td>
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<td>-4.4</td>
<td>2.1</td>
<td>-1.3</td>
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<tr>
<td>3²</td>
<td>2³</td>
<td>√72</td>
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<tr>
<td>66.6%</td>
<td>2/3</td>
<td>0.67</td>
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Another family of math talks

Like many mathematical objects, problems, and tasks, math talks often come in families. Some nice features:

• Intentional use of related questions can help students make connections or generalize to infer properties
• Imposing constraints can raise the level of challenge
• Choices can adjust content focus to fit curricular topics

One family of math talks was the **number of the day** that many folks have been using.

Another interesting family of math talks is **number lines**.
What is a reasonable value for the number at the arrow? Why?
Number Lines: Estimation

About where would 835 be? 212? 315? Justify your answers with mathematically convincing arguments.
Number Lines: Integers

Where would you place 6? -2? -9? Why?
What about 6 + -2? 6 – -2? Why?
What about 3 · 2? 3 · -2? -3 · -2? Why?
Number Lines: Decimals

Where would you place 0.46? 0.7?
0.51? 0.509? 0.59?
0.8? 0.08? 0.008?
Number Lines: Probability

Where would you find a “very likely” event?
Where would you find an “impossible” event?
Where is the probability that it’ll snow tomorrow?
The probability that it won’t snow tomorrow?
Number Lines: Further Extensions

It doesn’t stop here. Further extensions include:
- Radicals or exponents
- Solutions to a family of equations
- Solutions to inequalities
- Going to two variables: the coordinate plane
- Wrapping the number line around: unit circle
**Math Talk: Can you find...**

- Five numbers whose mean is 8.
- Five numbers whose mean is 8, **without** using 8 as one of the values.
- Five numbers whose mean is 8, but whose median is 2.
- Five numbers whose mean is 8, but whose median is *at least* 20.
Should you play this game?

Game Rules: Roll two standard number cubes.

• You win $1 if BOTH numbers are less than 5.
• You lose $1 if EITHER number is 5 or more.
Math Talk: A Snail’s Pace

You are stuck in bad traffic and can only travel 5 miles in 20 minutes. How far can you travel in an hour? In a half an hour?
Facilitator has a rule in their head.
Solicit inputs from participants.
Say and scribe the outputs.
Participants try to determine the rule.
If someone thinks they know the rule, pose an input to them to check.
Math Talk: Relational Thinking

\[7 + 6 = x + 5\]
\[43 + 28 = x + 42\]
\[28 + 32 = 27 + x\]
\[67 + 83 = x + 82\]
\[12 + 9 = 10 + 8 + x\]
\[345 + 576 = 342 + 574 + x\]
\[46 + 28 = 27 + 50 - x\]
Math Talk: Relational Thinking

3 \cdot ___ = 3 + 3 + 3 + 3 + 3 + 3 + 3

6 \cdot ___ = 6 + 6 + 6 + 6 + 6

8 \cdot 3 = 8 + 8 + ____

5 \cdot 4 = 4 + 4 + ____
Find the following $x$ (and $y$) values

\[
\begin{align*}
3x &= 0 \\
9x &= 0 \\
0 &= -27x \\
x - 8 &= 0 \\
x + 1 &= 0 \\
x + 546 &= 0 \\
2(x - 9) &= 0 \\
130(x - 15) &= 0 \\
6(x + 2) &= 0 \\
x y &= 0 \\
(x - 4)(x - 3) &= 0 \\
(x - 7)(x + 2) &= 0 \\
x(x - 1) &= 0
\end{align*}
\]
Many Varieties of Math Talks

• Number of the Day, other Mental Math
• Number Lines, other Spatial Visualization
• Sequences of Related Expressions
• Relational Thinking – Expressions
• What’s My Rule? – Functions
• Logical Reasoning across Domains
Math Talks can help develop

• Communication and explanation

• Logical thinking, reasoning, and arguments

• Student engagement

• Conceptual understanding, sense-making with connections

• Problem-solving

• Procedures through review and contextual practice
Math Talks can help develop

• Sense for numbers, relationships, operations
  – Intuition
  – Judgment
  – Foundation for fluency

• Fluency
  – Efficiency
  – Accuracy
  – Flexibility
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
A Grade Level Plan at a School

PLAN:
• What we are teaching next week?
• What are students struggling with?
• Select a math talk including a prompt that you as a team agree upon.
• Anticipate student responses
• Try it out in your classroom before the next meeting

SHARE:
• What were the successes/challenges?
• Would you have changed the prompt?
• What type of questions did you ask as the math talk progressed?
• How did your students react?
• How did it inform your future instruction?
• If you recorded bring your charts to share
At the School/District Level:
Try a Math Talk Jigsaw using these four prompts:

- 85% of 60
- 3 ⅓ of 18
- What’s my rule?
- Dot Pattern
Mechanics of a Math Talk Jigsaw

In like groups:

• Brainstorm the many different ways to solve your particular prompt
• Discuss
  – how to record “student” strategies and solutions.
  – how to handle potential pitfalls

Jigsaw:

• One at a time, pose your prompt to your peers
• Practice recording peer strategies and solutions

Next Steps:

• Try out a prompt in your classroom and share a team.
Sources and References

• A gold mine of examples: Secondary Number Sense Routines (San Diego) — http://goo.gl/MPp0y

• More resources on Math Perspectives, Math Solutions sites (search for Ruth Parker, Cathy Young)

• *Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5* (Parrish)

• Two great articles from *MTMS*: “Never Say Anything a Kid Can Say” and “Orchestrating Discussions”
Thanks

• Thanks to David Foster and the work of the Silicon Valley Mathematics Initiative (SVMI)
• Thanks to those who have created other useful resources
• Thanks to the teachers who have welcomed me and my colleagues into their classrooms through the South and West Cook Mathematics Initiatives
• And thanks to the Searle Funds at The Chicago Community Trust, the South Cook and West40 Intermediate Service Centers, and the UIC Learning Sciences Research Institute for providing support for SCMI and WCMI

And thank you!
Further Questions
Some questions to think about

Think about and discuss at your table:

• What answers might students give (that we aren’t likely to)?

• As a facilitator, what do you do with the answers (both correct and incorrect)?

• For learners, how did this contrast with a traditional bell-ringer?
Math Talk Outcomes

Teachers participating in our initiatives have been using math talks for a few years. They told us they are seeing (and we get to see too):

- Raised student engagement
- Demonstrates valuing student thinking
- Models methods for productive discourse
- Demands problem-solving, arguments
- Shows multiple methods and connections
How do we get Math Talks to work?

Math talks may have potential, but how do we get them to work in real classrooms?

What are some features that make math talks work well?

What are some possible challenges, concerns, or questions?
Potentially effective features

• Individual think time
• Finger signals make it safe
• Elicits multiple methods, not just right answer
• Students must talk to explain/justify method
• Students must make sense of others’ methods
• Scribing helps students follow what’s going on
• Visual representations can get kids talking
• Opportunities to connect and generalize
Potential challenges and concerns

• What if no one says anything?
• How do you handle wrong answers?
• What if students’ explanations are unclear?
• What if students don’t make connections?
• What is appropriate for teacher to say/add?
• How and when do you end the math talk?
• Can the math talk routine be modified? How?
• What if math talks are getting boring for kids?
Planning Math Talks *Intentionally*

Math talks can help develop students’ proficiency in making arguments and critiquing reasoning. Just like re-engagement lessons, math talks are most effective when you:

- Have a **goal** for your use of the math talk
- Have **anticipated** potential student responses

With a partner, think about a possible **goal** for a math talk, come up with a **prompt**, and think of possible student responses (correct or incorrect).
Math Talk Jigsaw

- Groups examine different problems
- Meet with all others who have your same problem to prepare for your presentation
- Present in mixed groups
- Return to your preparation group to debrief presentation and reflect
Math Talk Jigsaw

• In like groups, brainstorm as many different strategies and solution paths you can come up with for your particular problem.
• Share ideas about how you will record “student” strategies and solutions.
• Share ideas about how to handle potential pitfalls in the presentation and recording of your particular problem.
Math Talk Jigsaw Prompts

• 35% of 120

• $3\frac{1}{3} \times 18$

• What’s My Rule
Devising Our Own Math Talks

• Think of an important idea or a common issue your students face in terms of conceptual understanding or fluency

• Think of a possible prompt for a math talk for that idea or issue

• Run your prompt by your partner and brainstorm possible student responses
Some Math Talk Techniques

• Individual think time
• Signals
• Organized scribing and juxtaposition
• Pair and share
• Whole-group share
• $3 \times 5$ card follow-up question
• Continuation or extension over days or a week
• Written formative assessment: multiple strategies
Some Math Talk Tools

- Overhead transparencies
- Charts
- Butcher paper recordings
- White boards
- Document camera
- Smart boards
- 3 × 5 cards
- Daily logs
Some Math Talk Tips

• Let students practice similar problems and routines
• Create a safe environment and encourage sharing
• Meet their needs: ways of sharing, ramp of difficulty
• Start where they are, have a goal, plan sequences
• Press students to make sense, clarify thinking
• Press students to explain and justify methods
• Purposefully choose, sequence strategies & sharers
• Record thoughtfully
• Ask questions to help students connect methods
• It’s the math (not answer or variety of strategies)
• Keep them short
Integrating Math Talks

• How do you think math talks can benefit your students?

• What are some issues you might face as you try math talks?

• What will you try in your classroom in the next ...week? ...month? ...year?