



**WELCOME TO**  
*Learning Progressions of  
Elementary Data and  
Measurement*



# K - 3 Categorical Data



Grade	Categorical Data Standard	Notable Connections
K	<p><b>K.MD.3.</b>  <i>Classify</i> objects into given categories, <i>count</i> the number of objects in each category and sort (order the categories) the categories by count. Limit category counts to be less than or equal to 10.</p>	<p><b>K.CC.</b>  <i>Counting to tell the number of objects</i></p> <p><b>K.CC.</b>  <i>Comparing numbers</i></p>
1	<p><b>1.MD.4.</b>  <i>Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.</i></p>	<p><b>1.OA.</b>  <i>Problems involving addition and subtraction</i>  - Put-together, take-apart, compare  - Problems that call for addition of three whole numbers</p>
2	<p><b>2.MD.10.</b>  <i>Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.</i></p>	<p><b>2.OA.</b>  <i>Problems involving addition and subtraction</i>  - Put-together, take-apart, compare</p>
3	<p><b>3.MD.3.</b>  <i>Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.</i></p>	<p><b>3.OA.3</b>  <i>Problems involving multiplication</i></p> <p><b>3.OA.8</b>  <i>Two-step problems using the four operations</i></p> <p><b>3.G.1</b>  <i>Categories of shapes</i></p>

# Kindergarten

Work with data uses counting and order relations.

- *Classify objects into categories*
- *Count number of items in each category*
- *Make “more than” or “less than” comparisons*



# Grade 1

Students in Grade 1 begin to organize and represent categorical data.

- *Identify an attribute with which to sort objects*
- *Arranging of objects in piles is mirrored to arranging of marks into groups*
- *Students can ask and answer questions about categorical data based on a representation of that data.*



Ocean	Farm	Wild	House
6	3	6	2

1. Farm + Ocean = \_\_\_\_\_

+

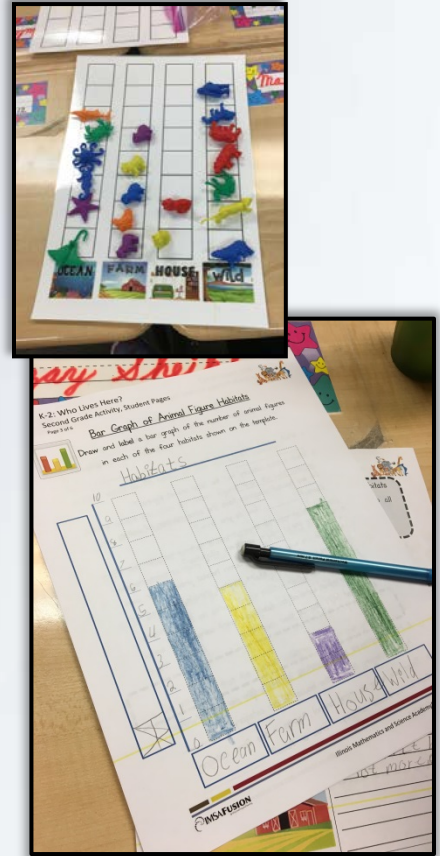
6. Ocean - House = \_\_\_\_\_

-

# Grade 2

Students in Grade 2 draw a picture graph and a bar graph to represent a data set with up to four categories.

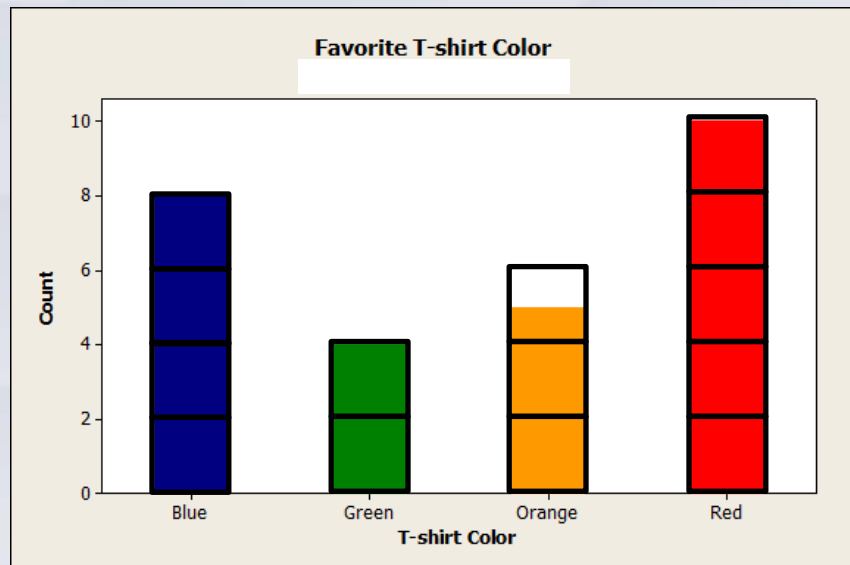
- *Students can solve simple put-together, take-apart, and compare problems using information presented in a bar graph.*
- *The horizontal axis is not a scale of any kind; position along the horizontal axis has no numerical meaning.*
- *The vertical axis is the “count scale” for the graph*
- *Count scale in a bar graph is a segment of a number line diagram*









# Grade 3

Students in Grade 3 now draw picture graphs in which each picture represents more than one object and bar graphs in which the height of each bar must be multiplied by the scale factor.

- *Connects with Grade 3 emphasis on multiplication*
- *Focus on gathering categorical data in an authentic context*



Cell Phone Sales	
Google Pixel	
Apple iPhone	
Samsung Galaxy	
Motorola Droid	
LG	
 = 5 cell phones	

# Where the Categorical Data Progression is heading

Work with categorical data in the early grades leads to later work in eighth-grade with bivariate categorical data (categorized by two attributes) and two-way tables.

	Lived	Died
Male	367	1364
Female	344	126

Lead into a discussion of joint and marginal probabilities





## 2 – 5 Measurement Data



Grade	Measurement Data Standard	Notable Connections
2	<p><b>2.MD.9.</b>  <i>Generate measurement data by measuring <b>lengths</b> of several objects to the <b>nearest whole</b> unit, or by making <b>repeated measurements</b> of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.</i></p>	<p><b>1.MD.2.</b>  <i>Length measurement</i></p> <p><b>2.MD.6.</b>  <i>Number line</i></p>
3	<p><b>3.MD.4.</b>  <i>Generate measurement data by measuring lengths using rulers marked with <b>halves</b> and <b>fourths</b> of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.</i></p>	<p><b>3.NF.2.</b>  <i>Fractions on a number line</i></p>
4	<p><b>4.MD.4.</b>  <i>Make a line plot to display a data set of measurements in <b>fractions</b> of a unit (<math>\frac{1}{2}, \frac{1}{4}, \frac{1}{8}</math>). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p>	<p><b>4.NF.3,4.</b>  <i>Problems involving fraction arithmetic</i></p>
5	<p><b>5.MD.2.</b>  <i>Make a line plot to display a data set of measurements in <b>fractions</b> of a unit (<math>\frac{1}{2}, \frac{1}{4}, \frac{1}{8}</math>). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount each beaker would contain if the total amount in all the beakers were redistributed equally.</i></p>	<p><b>5.NF.1,2,4,6,7.</b>  <i>Problems involving fraction arithmetic</i></p>

# What is measurement?

ACTIVITY: *Measure the bucket!*



1. Decide on an attribute to be measured.
2. Select a unit that has that attribute.
3. Compare the units – by filling, covering, matching, or using some other method – with the attribute of the object being measured. The number of units required to match the object is the measure.

-Van de Walle, p. 376

*Measurement* is the process of assigning a number to a magnitude of some attribute shared by a class of objects.

# Measurement Instruction

## STEP ONE – Making Comparisons

**Goal:** Students will understand the attribute to be measured.

**Type of Activity:** Make comparisons based on the attribute.

## STEP TWO – Using Models of Measuring Units

**Goal:** Students will understand how filling, covering, matching, or making other comparisons of an attribute with measuring units produces a number called a measure.

**Type of Activity:** Use physical models of measuring units to fill, cover, match, or make the desired comparison of the attribute with the unit.

## STEP THREE – Using Measuring Instruments

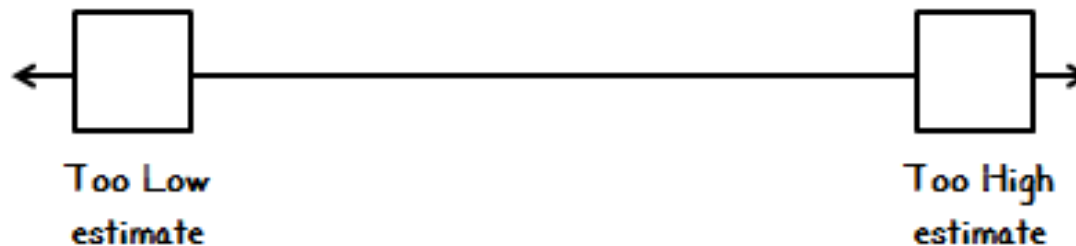
**Goal:** Students will use common measuring tools with understanding and flexibility.

**Type of Activity:** Make measuring instruments and use them in comparison with the actual unit models to see how the measurement tool is performing the same function as the individual units. Be certain to make direct comparisons between the student-made tools and the standard tools.

# Estimating and Benchmarks

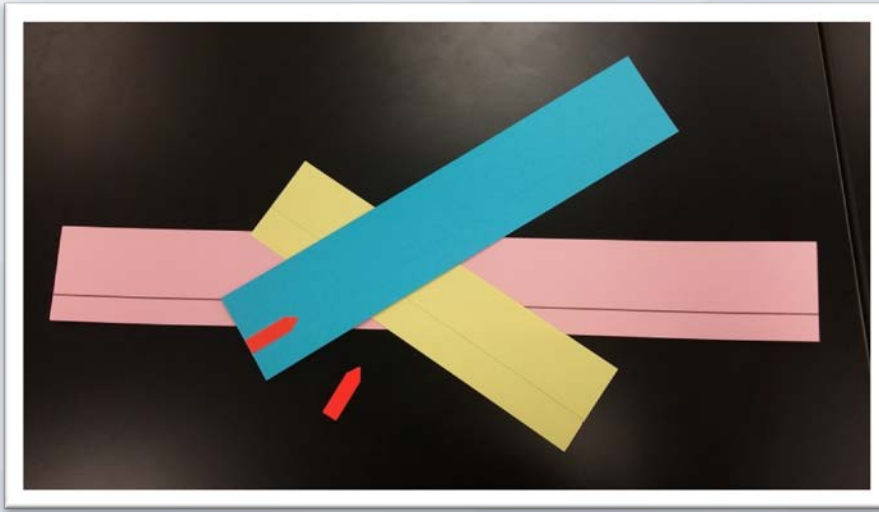
Always begin a measurement activity with students making an estimate, even if using non-standard units.

- ✓ Develop and use benchmarks or referents for important units.
- ✓ Use “chunking” when appropriate.
- ✓ Use subdivisions.
- ✓ Iterate a unit mentally or physically.

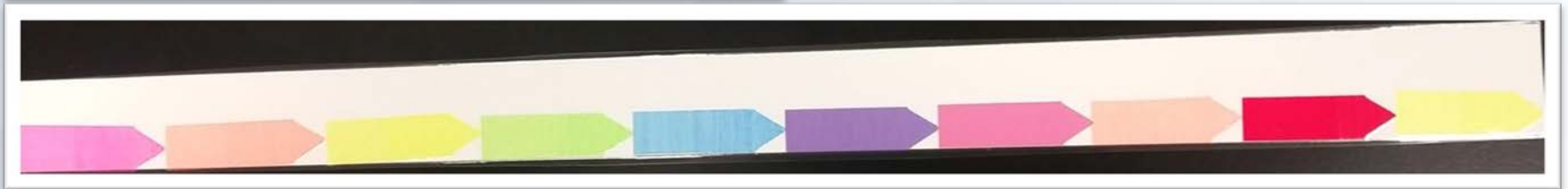


# What is length?

ACTIVITY: *Estimate and Measure*



1. Choose a unit of measure.
2. Subdivide (mentally and physically) the object by that unit, placing that unit end to end (tiling or iterating) alongside that unit without gaps or overlaps.



*Length* is a characteristic of an object found by quantifying how far it is between the endpoints of the object.

What are four important principles of iterating units?

# Concepts and Skills

Length-unit iteration

Connecting measurement with physical units and with a ruler

Meaning of numerals on the ruler

Accumulation of distance

Alignment of zero-point

### Kindergarten

- ✓ Describe and compare measurable attributes
- ✓ Directly compare two objects with a measurable attribute in common

### Grade 2

- ✓ Measure and estimate lengths in standard units with rulers and length units
- ✓ Draw simple unit rulers
- ✓ Units must be the same length
- ✓ Learn inverse relationship between the size of the unit of length and the number of units requires to cover a specific length or distance.

### Grade 1

- ✓ Continue to use direct comparison
- ✓ Should be able to use indirect comparison, drawing on transitivity (using a third object to compare two objects)
- ✓ Seriation (order a set of object by length)
- ✓ Measure lengths indirectly and by iterating length units

### Grade 3

- ✓ Focus is on solving real-world and mathematical problems involving perimeters of polygons.
- ✓ Begin with counting unit lengths and progress to finding faster ways to find the perimeter length than just adding lengths
- ✓ Students learn to subdivide length units



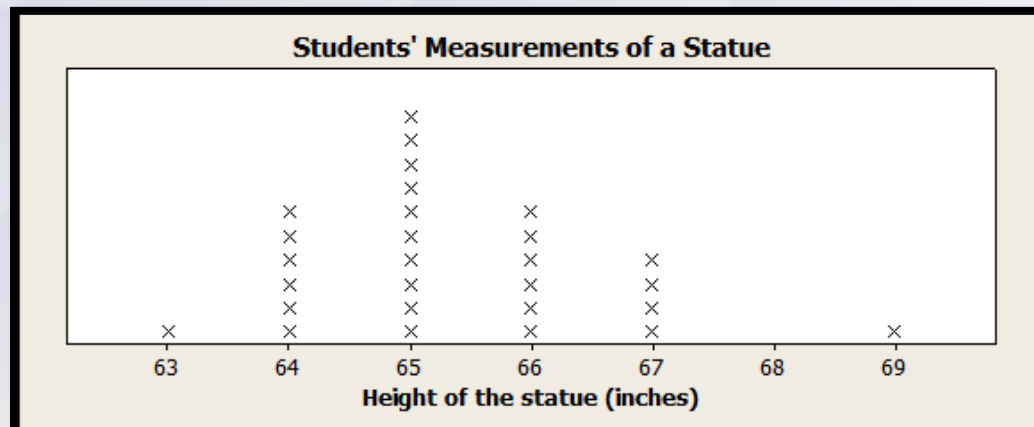
# Measurement Data: Grade 2

Students in Grade 2 measure lengths to generate a set of measurement data.

- *Students can be presented with items to measure or can generate their own ideas about what to measure*
- *A display of measurement data must present the measured values with appropriate magnitude and spacing*
- *Construct a line plot with a number line diagram on the horizontal axis*

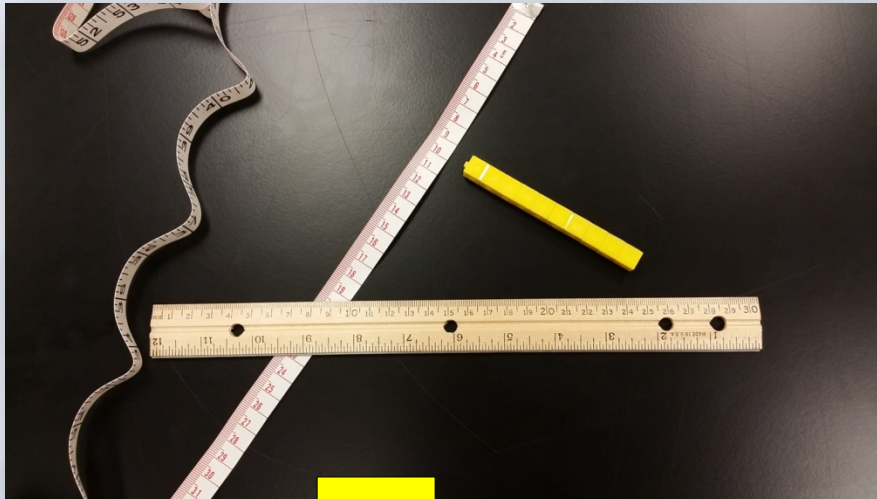
MP.6

*Measuring and recording data require attention to precision*



# Measurement Data

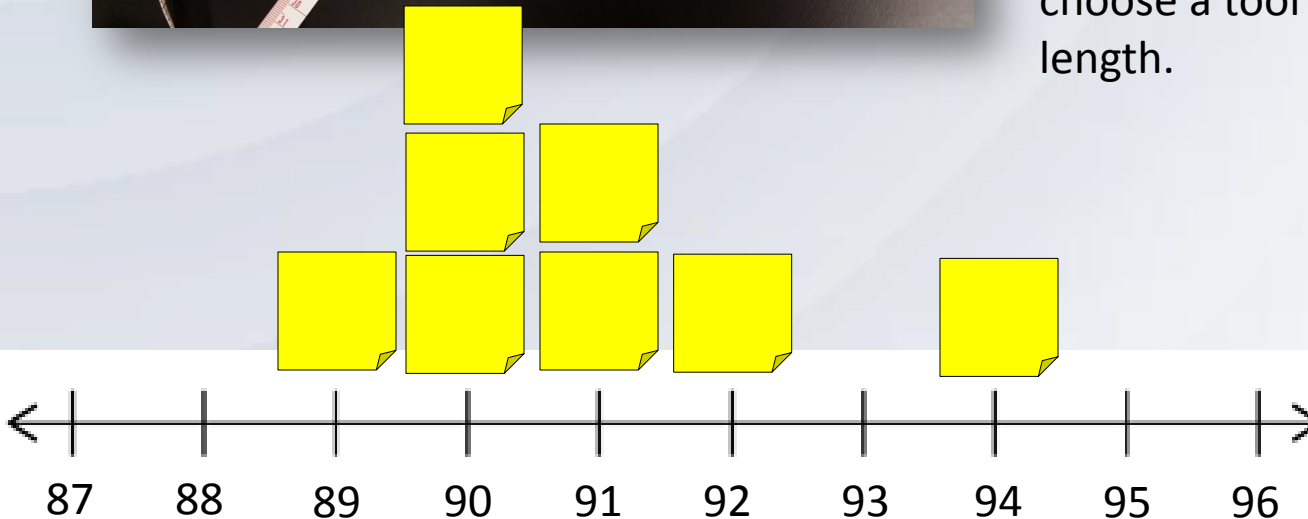
ACTIVITY: *Using different measurement tools*



We will be measuring a wooden dowel rod using various tools that measure in centimeters.

First, let's estimate the length of the wooden dowel rod.

Next, each person at the table will choose a tool to measure the length.

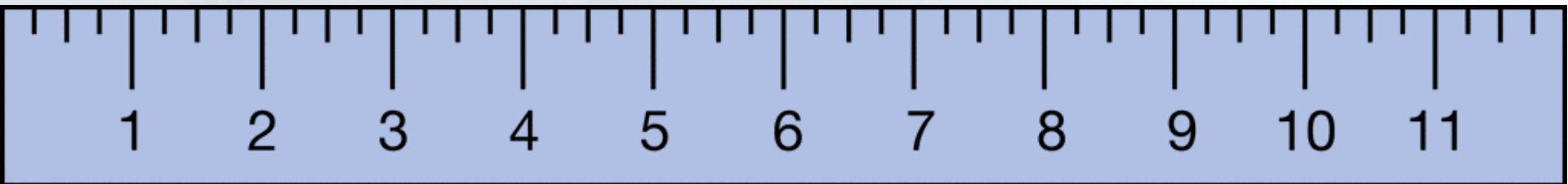


- How is the number line diagram constructed?
- How can each value of the data set be represented on the line plot?
- Are there any interesting visual features of the line plot? (not part of standard)
- What questions can we ask about the data?

# Grade 3

Students in Grade 3 also measure lengths to generate a set of measurement data.

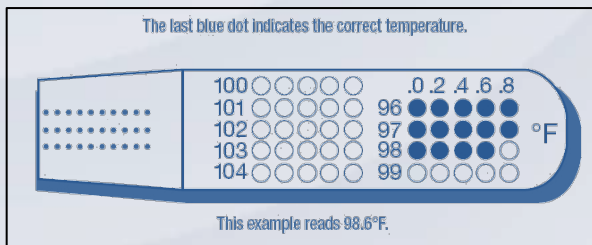
- *Students use rulers marked with halves and fourths of an inch to work with data involving fractional measurement values.*
- *Look at extreme values to construct a number line diagram*
- *Questions posed are based upon data presented in line plots*



# Grades 4 and 5

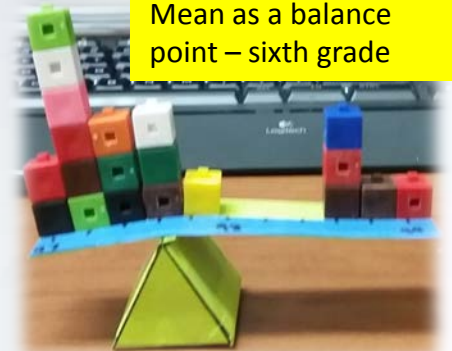
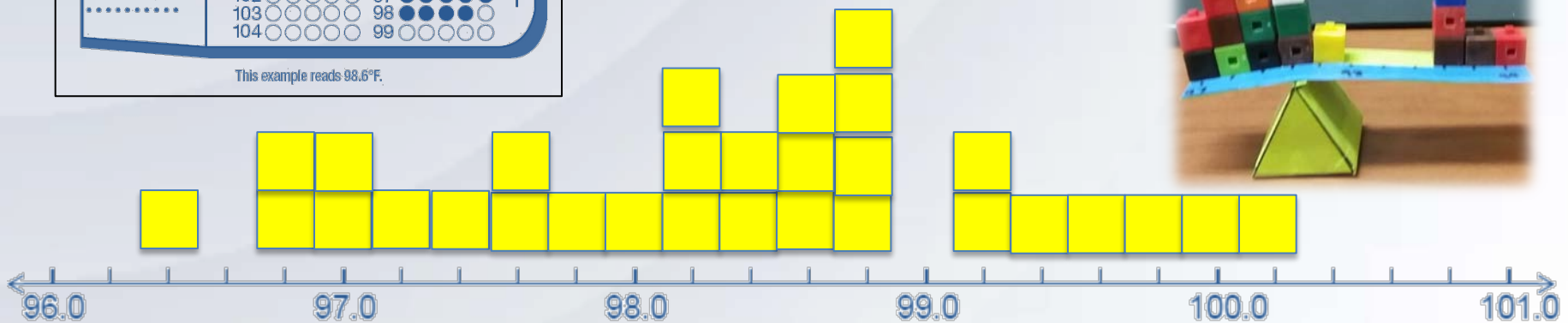
Students in Grades 4 and 5 measure lengths to generate a set of fractional measurement data.

- *Still using line plots, students may need to label measurement scale using a common denominator*
- *Decimal data can also be used in this grade*
- *Work with data in science and other subjects*



Mean as equal distribution –  
informal discussion in fifth grade

Mean as a balance  
point – sixth grade



# Measurement Data

ACTIVITY: *Arm span data*



**Discuss:**

*In what ways can we use this collected data?*

We will be measuring the length of your arm span using inches (to the nearest  $\frac{1}{8}$  of an inch).

Working with a partner, tape one end of a strip of adding machine paper to the tip of your longest finger.

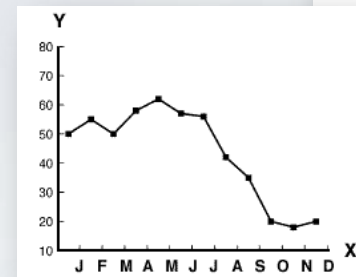
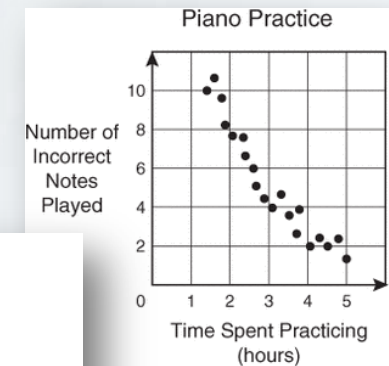
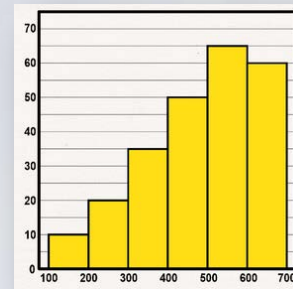
Next, stretch out your arms and have your partner unroll the adding machine paper slightly past your outstretched arms and rip off the roll.

Fold the ripped edge of the adding machine paper to the tip of the longest finger.

Remove and label the paper strip with your name.

# Where the Measurement Data Progression is heading

- ✓ At the end of Grade 5, students should be comfortable making line plots for measurement data and analyzing those line plots.
- ✓ These line plots are developed into histograms in sixth grade (univariate measurement data). Histograms have a measurement scale and a count scale similar to line plots.
- ✓ The other evolution of line plots involves graphing bivariate measurement data using two measurement scales (coordinate plane). Line graphs and scatter plots are the representations used.





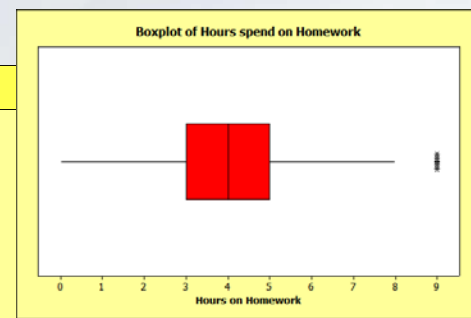
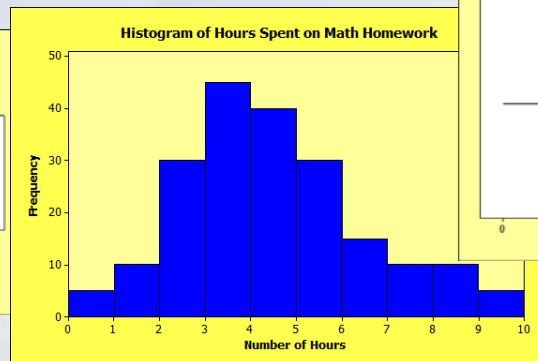
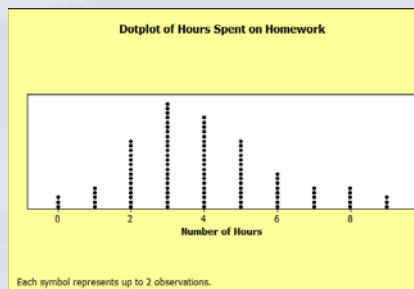
# 6 – 8 Statistics and Probability





# Grade 6

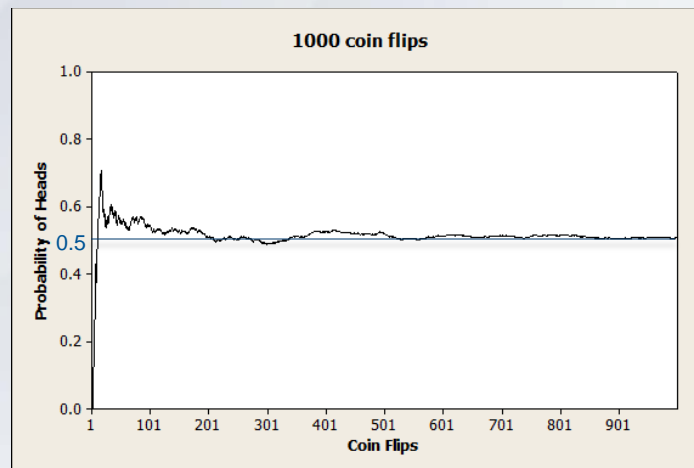
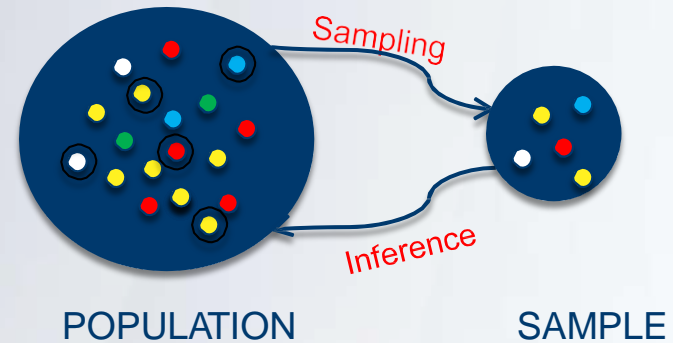
- Students in Grade 6 develop a deeper understanding of variability.
- Descriptions of data distributions are more precise, and numeric measures of center and spread are applied to quantitative data.
- Statistical reasoning is developed.



Variable	N	Mean	Minimum	Q1	Median	Q3	Maximum
C1	200	4.000	0.000	3.000	4.000	5.000	9.000

# Grade 7

- Focus shifts to production of data: random sampling and making inferences
- Investigating probability as long-run relative frequencies
- Observe connections between theoretical and empirical models

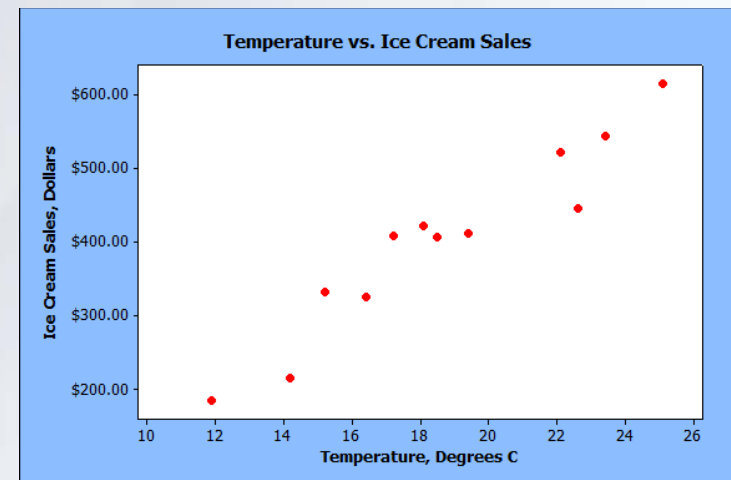


# Grade 8

- **Bivariate Quantitative Data**
  - Scatter Plots
  - Introduction to Linear Regression
- **Bivariate Categorical Data**
  - Two-way tables
  - (Segmented Bar Graphs)

Do eighth-grade students at this school prefer chocolate chip cookies or peanut butter cookies?

	Girls	Boys	Total
Chocolate Chip	25	40	65
Peanut Butter	32	23	55
Total	57	63	120



# High School



- Standard deviation is introduced to expand upon students' abilities to describe and compare data distributions
- Knowledge of probability is expanded to include conditional probability and independence
- Students move beyond analyzing data to making sound statistical decisions based on probability models
- Students understand that probability is enmeshed with data analysis and use that knowledge to make inferences on data collected from surveys and experiments.

Who Lives Here? K – 2 Categorical Data Lesson

[https://digitalcommons.imsa.edu/pfs\\_pr/13/](https://digitalcommons.imsa.edu/pfs_pr/13/)

Van de Walle, J. A., Karp, K., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: teaching developmentally*. NY, NY: Pearson. Professional Development Edition.

Progressions Documents for the Common Core State Standards for Mathematics. (n.d.). Retrieved February 27, 2018, from <https://achievethecore.org/page/254/progressions-documents-for-the-common-core-state-standards-for-mathematics>

Measurement & Data (data part): Grades K - 5

**THANK  
YOU**

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