NOTES

Logistics: This lesson is intended for students in Grades 6-8 in an introductory pre-algebra or geometry course as a hands-on investigation of geometric transformations. This specific lesson focuses on students exploring the relationship between two figures on the coordinate plane. Additionally, students will create a personal definition about the mathematical concept called, "dilation".

Materials:

per student:

1 – Copy of Student Pages

per pair of students:

- *1 – Set of rectangles (5 rectangles)*
- *1 – 11"X17" Coordinate grid board*
- *Writing utensils*

Time: One 45-50 minute class period

Objectives/Standards:

Manipulating similar rectangles, students will determine the ratio of various side lengths in an exploration of geometric transformations; specifically, **dilations**. They will identify the effect of dilation about the origin on the rectangle's coordinates and explore the similarities between the original figure (**pre-image**) and the transformed figure (**image**). Additionally, students will determine a mathematical expression that represents a dilation about the origin on the coordinate plane.

- **CCSS.MATH.CONTENT.7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- **CCSS.MATH.CONTENT.8.G.A.3** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- **CCSS.MATH.CONTENT.8.G.A.4** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- **CCSS.MATH.CONTENT.5.G.A.2** Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Enlarging Transformations

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Introduction: A transformation is a mathematical term that describes a specific way to manipulate the shape of a point, a line, or a shape. In geometry, there are two types of transformations: isometric (rigid) and non-isometric. Translations, reflections and rotations maintain congruency and are therefore isometric.

On the other hand, a dilation is a non-isometric transformation. A dilation is a transformation in which a figure maintains its original shape, but its size is either enlarged or reduced. This type of transformation produces an image that is similar to its pre-image.

In order for a dilation to take place the following components must be included: a scale factor, or ratio of dilation, and the center of the dilation, or the fixed point on the plane about which all the points are contracted or expanded.

The algebraic notation for a dilation: $D_k(x, y) = (kx,\ ky)$

- Where D is the center of dilation at the origin and k is the scale factor.
- The image created by a dilation is either an enlargement or a reduction according to the k value.
- An enlargement is a dilation used to create an image larger than the pre-image when $k > 1$
- A reduction is a dilation used to create an image smaller than the pre-image when $0 < k < 1$
- If $k = 1$, the image and pre-image are congruent.

NOTE: This notation only applies when the center of dilation is located at the origin.

Advanced Preparation

Prior to beginning the lesson, prepare the coordinate plane (Quadrant I only) paper. It is suggested that the template be printed on 1 centimeter 11"x17" grid paper with at least 25 units on the y-axis and at least 42 units on the x-axis. Each group of students will also need one set of six rectangles.

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Refer to the following dimensions when constructing the rectangle sets:

Rectangle	Dimensions	Rectangle	Dimensions
#1	5cm X 8cm	#4	2.5cm X 4cm
#2	10cm X 16cm	#5	10cm X 13cm
#3	25cm X 40cm		

Activity:

Students should be placed in groups of 2 or 3 for this activity.

Provide each team with one 11"X17" coordinate plane, student page packet, and set of rectangles. At this time, model to students how to maneuver the coordinate plane and use it as a measurement tool.

Prior to beginning the exploration, you may choose to explore the students' understanding of plotting points and identifying locations on the coordinate plane. It may be necessary to ask the students to use their finger to identify various points on the coordinate plane.

Next, students will explore the mathematical relationship between several rectangles by placing them on the coordinate plane and using the grid as a measuring tool. Students will compare the side length dimensions and coordinate values from the original rectangle (pre-image) to the other rectangles (image).

First, students will place Rectangle #1 on the coordinate plane with vertex "A" at the origin. Students will then identify the corresponding coordinates of each vertex, record the side length dimensions of the rectangle, and quantify the mathematical relationship (ratio) between the length and width. Students will repeat this process for each additional set of rectangles described in their student pages.

The student pages will guide learners to observe several relationships between the two rectangles analyzed in each set. Identifying characteristics of a dilation, students will first record how many times larger/smaller one rectangle is compared to the other. This information leads students to the understanding of **scale factor.** Next, students will compare the coordinates, dimensions and shape of the selected rectangles. During this time, students should begin to recognize the mathematical relationship between

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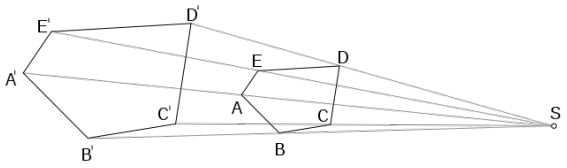

the coordinate points, along with the idea that although the rectangles are not the same size, they are the same shape (similarity). These are essential ideas in understanding that a dilation is a non-isometric transformation.

During this exercise, monitor students and provide help as necessary. Allow sufficient time for the students to complete every step. Throughout the activity, prompt students to make observations and find patterns between data values.

After collecting data, students will answer a series of questions designed to solidify their understanding of dilations. Students should complete these questions within their small group. When all students are finished, review the questions in a whole class setting:

- **How do changes in the lengths of the sides relate to the changes in the coordinates?** [*The side lengths and coordinates change at the same rate. If the side lengths double, the x- and y-values also double.]*
- **Review the scale factor of each set of rectangles.** [*Note: There is no scale factor to describe the relationship between Rectangle #1 and Rectangle #5. These are not similar rectangles, and therefore do not have a scale factor.*]
- **According to your data, Rectangles #1, #2, #3 and #4 represent dilations on the coordinate plane. Based on your observations, what is necessary for two objects to be dilations?** [*A dilation is a geometrical transformation that retains the shape of the original figure but not the size. It must have a scale factor and a center of dilation.*]
- **Develop a mathematical expression that reflects the relationship between the scale factor, *k*, and the figure's coordinates, *(x, y)*.** [$D_k(x, y) = (kx,\ ky)$. *Note: Students may investigate if this rule always works by testing the expression using various centers of dilation.*]
- **Create your own definition of a dilation based on your observations and data.** [*Student answers will vary. Appropriate definitions may include use of scale factor, enlargement, reduction, similarity, size, shape and/or center of dilation.*]

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Allow students time to share their definitions with one another during a class discussion. After, pose the following debrief questions and elicit answers from student volunteers.

Debrief Activity:

- ***In what ways do dilations differ from translations, reflections, and rotations? In what ways are they the same?*** [*Translations, reflections, rotations and dilations are all performed on the coordinate plane and maintain the shape of the original figure. However, unlike a translation, reflection and rotation, the dilation of a shape does not result in a congruent image, but instead, a similar figure. Therefore, a dilation is a non-isometric transformation.*]
- ***How do you determine if a transformation of two figures represents a dilation?*** [*When the image has the same shape as the pre-image, but it is larger or smaller according to the scale factor, the transformation is a dilation.*]
- ***What is the effect of using a scale factor,*** k***, such that*** $0<k<1$ ***or*** $k>1$***?*** [*If* ***k*** *is larger than 1 the image will be larger than the pre-image, and if* ***k*** *is between 0 and 1 the image will be smaller than the pre-image.*]

EXTENSION:

To extend this lesson, students could apply their knowledge of geometric transformations to create their own images on a coordinate plane. Included in this activity are four additional exercises that require students to use translations, reflections, rotations and dilations to complete the "missing part" of the picture. By first identifying various points within the picture, students will complete a series of directions to transform each coordinate. This activity may take some time.

Additionally, students may explore the following concepts:

- the relationship between area and perimeter of the pre-image and the image after dilation, and how this relates to scale factor.
- the purpose of applying medication to the eyes and how the pupil dilates.
- the relationship between the lenses on a camera and their ability to focus on an object at various distances.

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CONCLUSION:

Dilation is a non-isometric transformation that produces an image that is the same shape as the original (pre-image), but is a different size. Figures that are dilated are not congruent, but similar. A dilation also has a center of dilation and a scale factor. The center of dilation is a fixed point in the plane in which all points are expanded or contracted.

The algebraic notation of a dilation:

$$\boldsymbol{D_k(x, y) = (kx,\ ky)}$$

- Where $\boldsymbol{D}$ is the center of dilation at the origin
- $\boldsymbol{k}$ is the scale factor
- If the scale factor is greater than 1, the image is an enlargement
- If the scale factor is greater than 0 and less than 1, the image is a reduction

RESOURCES:

Connecting Transformational Geometry and Transformation of Functions. (n.d.). Retrieved February 18, 2016, from https://pcmi.ias.edu/files/Final Connecting Geometry Transformations and Functions 10_20.pdf

Dilations. (2012). Retrieved February 18, 2016, from http://www.regentsprep.org/regents/math/geometry/gt3/ldilate2.htm

How To Perform A Dilation. (2016). Retrieved February 18, 2016, from https://www.khanacademy.org/math/geometry/transformations/dilations-scaling/v/dilating-from-an-arbitrary-point-example

ENLARGING TRANSFORMATIONS

Have you ever had eye drops applied to your eyes during an exam? Were you not able to see clearly for a period of time? Have you ever noticed what happens to your pupils when you are coming out of the dark and into the light? This is caused by your pupils attempting to focus. In mathematics, a geometric transformation called a dilation performs a similar action. You will now have the opportunity to explore this transformation by investigating various images on the coordinate plane.

Problem: *Explore and define the mathematical relationship between the side lengths, coordinate points, and shape of two rectangles.*

Materials per group:

- 1 – Set of rectangles
- 1 – 11”x17” 1cm coordinate grid paper
- Writing utensils

Procedure:

1.) Obtain all your materials from your teacher.
2.) Select Rectangle #1.
3.) Place the rectangle on the coordinate plane with vertex “A” at the origin.
4.) Identify the coordinates of each vertex.
5.) Identify the length and width of the rectangle. Then, determine the ratio of the dimensions.

Vertex	Coordinate
A	(,)
B	(,)
C	(,)
D	(,)

Length (L)	Width (W)
Ratio L/W:	

6.) Select Rectangle #2.
7.) Place the rectangle on the coordinate plane with vertex "A" at the origin
8.) Identify the coordinates of each vertex.
9.) Identify the length and width of the rectangle. Then, determine the ratio of the dimensions.

Rectangle 1			Rectangle 2	
Vertex	Coordinate		Vertex	Coordinate
A	(,)		A	(,)
B	(,)		B	(,)
C	(,)		C	(,)
D	(,)		D	(,)

Length (L)	Width (W)		Length (L)	Width (W)

Ratio L/W:		Ratio L/W:

How many times larger/smaller is Rectangle #2 compared to Rectangle #1?

Compare the coordinates, dimensions, and side length ratio between Rectangle #1 and #2. What do you notice? Has the shape changed? Use evidence to support your claim.

10.) Select Rectangle #3.
11.) Place the rectangle on the coordinate plane with vertex "A" at the origin.
12.) Identify the coordinates of each vertex.
13.) Identify the length and width of the rectangle. Then, determine the ratio of the dimensions.

Rectangle 1			Rectangle 3	
Vertex	Coordinate		Vertex	Coordinate
A	(,)		A	(,)
B	(,)		B	(,)
C	(,)		C	(,)
D	(,)		D	(,)

Length (L)	Width (W)		Length (L)	Width (W)

Ratio L/W:		Ratio L/W:

How many times larger/smaller is Rectangle #3 compared to Rectangle #1?

Compare the coordinates, dimensions, and side length ratio between Rectangle #1 and #3. What do you notice? Has the shape changed? Use evidence to support your claim.

14.) Select Rectangle #4.
15.) Place the rectangle on the coordinate plane with vertex "A" at the origin.
16.) Identify the coordinates of each vertex.
17.) Identify the length and width of the rectangle. Then, determine the ratio of the dimensions.

Rectangle 1			Rectangle 4	
Vertex	Coordinate		Vertex	Coordinate
A	(,)		A	(,)
B	(,)		B	(,)
C	(,)		C	(,)
D	(,)		D	(,)

Length (L)	Width (W)		Length (L)	Width (W)

Ratio L/W:		Ratio L/W:

How many times larger/smaller is Rectangle #4 compared to Rectangle #1?

Compare the coordinates, dimensions, and side length ratio between Rectangle #1 and #4. What do you notice? Has the shape changed? Use evidence to support your claim.

18.) Select Rectangle #5.
19.) Place the rectangle on the coordinate plane with vertex "A" at the origin.
20.) Identify the coordinates of each vertex.
21.) Identify the length and width of the rectangle. Then, determine the ratio of the dimensions.

Rectangle 1			Rectangle 5	
Vertex	Coordinate		Vertex	Coordinate
A	(,)		A	(,)
B	(,)		B	(,)
C	(,)		C	(,)
D	(,)		D	(,)

Length (L)	Width (W)		Length (L)	Width (W)

Ratio L/W		Ratio L/W

How many times larger/smaller is Rectangle #5 compared to Rectangle #1?

Compare the coordinates, dimensions, and side length ratio between Rectangle #1 and #5. What do you notice? Has the shape changed? Use evidence to support your claim.

How do changes in the lengths of the sides relate to the changes in the coordinates?

The ratio, or amount that an image is enlarged or reduced is, called the **scale factor**. In other words, this quantity identifies how many times larger/smaller the original figure is compared to its dilated image. Identify the scale factor between the following rectangles:

Rectangles	Scale Factor
1-2	
1-3	
1-4	
1-5	

According to your data, Rectangles #1, #2, #3 and #4 represent dilations on the coordinate plane. Based on your observations, what is necessary for two objects to be dilations?

Develop a mathematical expression that reflects the relationship between the scale factor, ***k***, and the figure's coordinates, ***(x, y)***.

Create your own definition of a dilation based on your observations and data.

Debrief Questions

Complete the following questions with a partner.

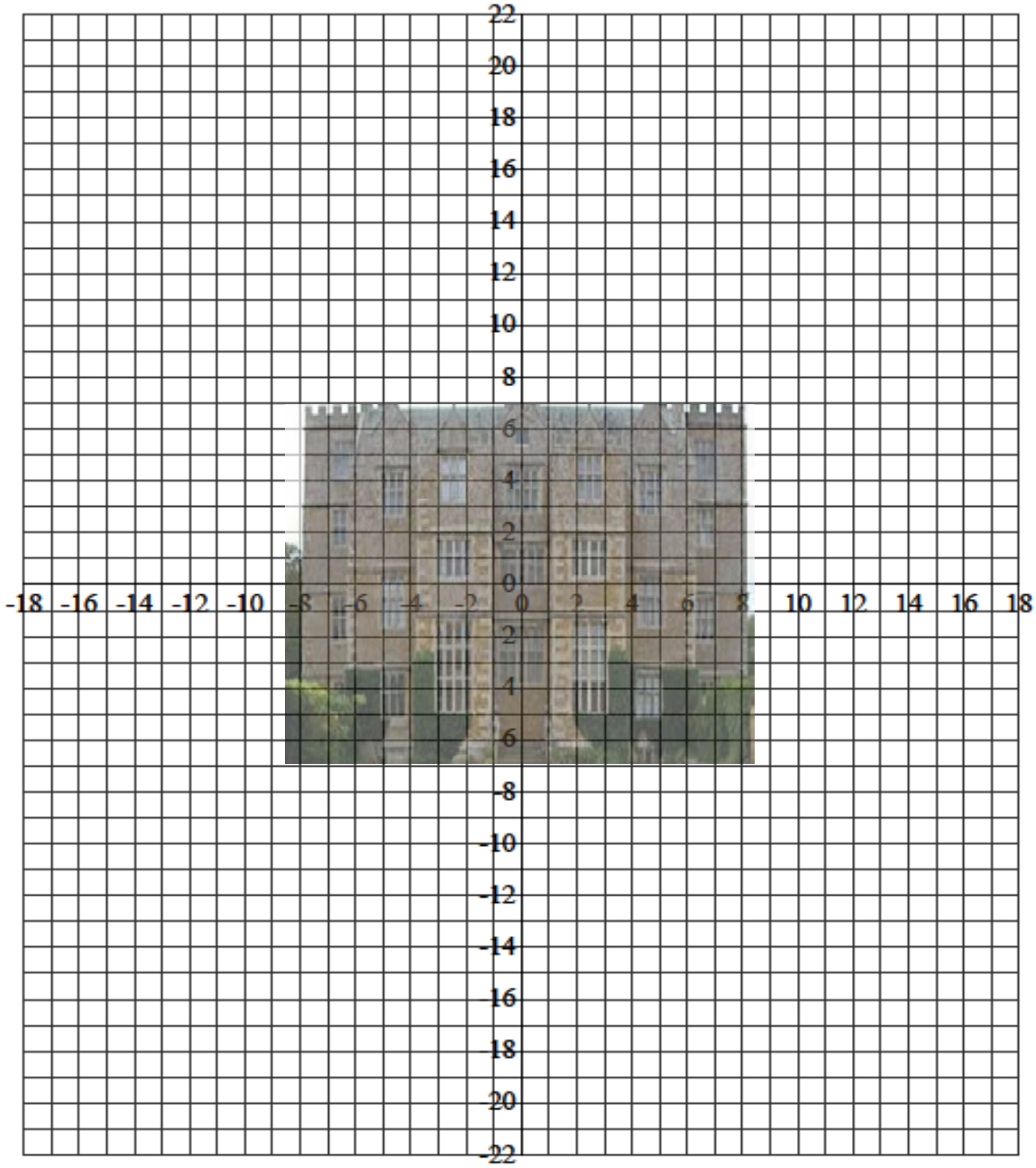
- In what ways do dilations differ from translations, reflections, and rotations? In what ways are they the same?

- How can you determine if a transformation of two figures represents a dilation?

- What is the effect of using a scale factor, k, such that $0<k<1$ or $k>1$?

DILATION

Perform a dilation with a scale factor of 4 and the center of dilation at the origin



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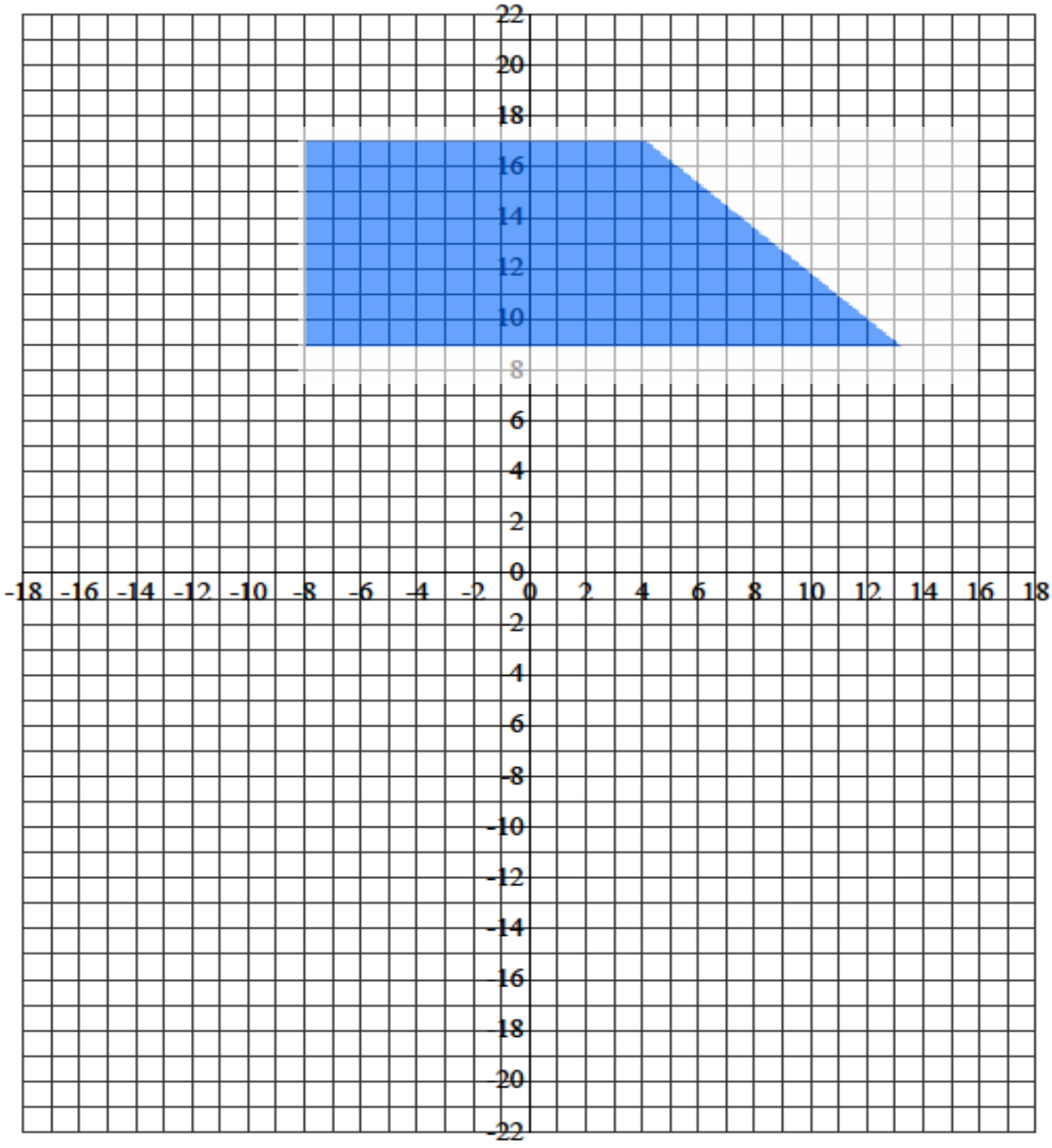
ROTATION

Rotate the image 90°, 180°, and 270° clockwise about the origin. Identify the popular logo.



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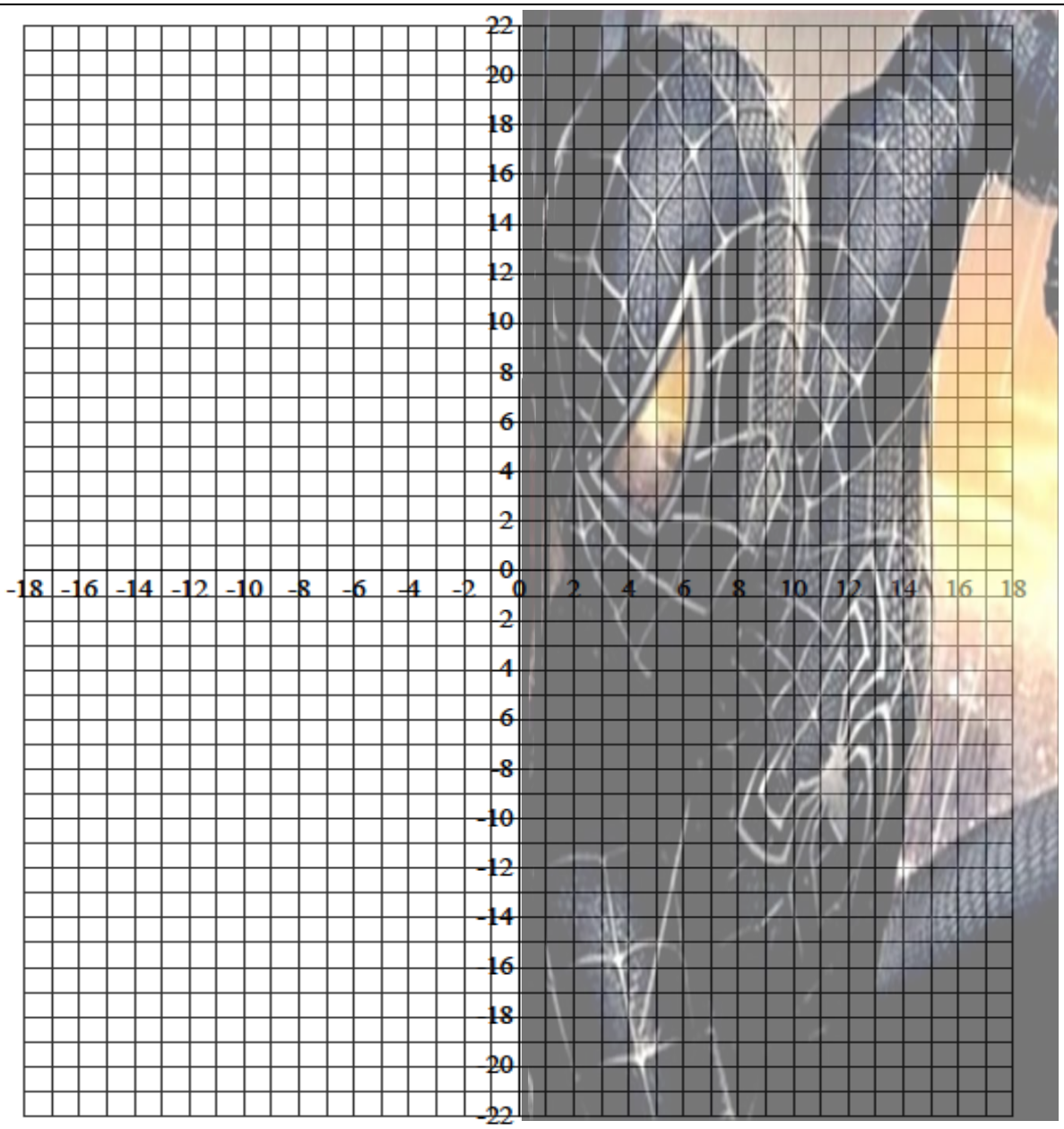
REFLECTION

Reflect the image across the y-axis



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TRANSLATION

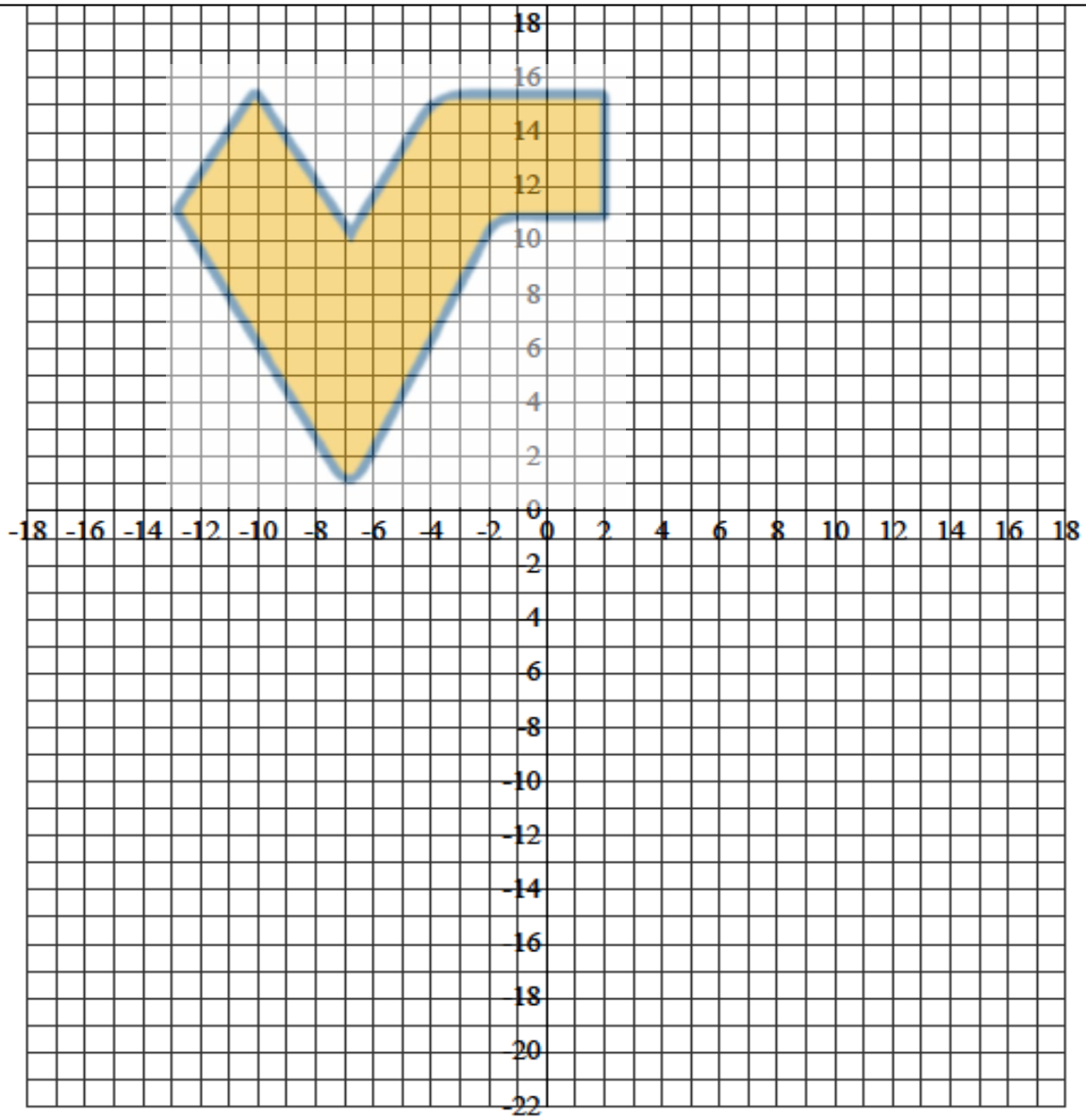
1) Translate the image 9 units to the right

2) Trace the image again

3) Translate the image 9 units down

4) Trace the image again

5) Identify the popular logo



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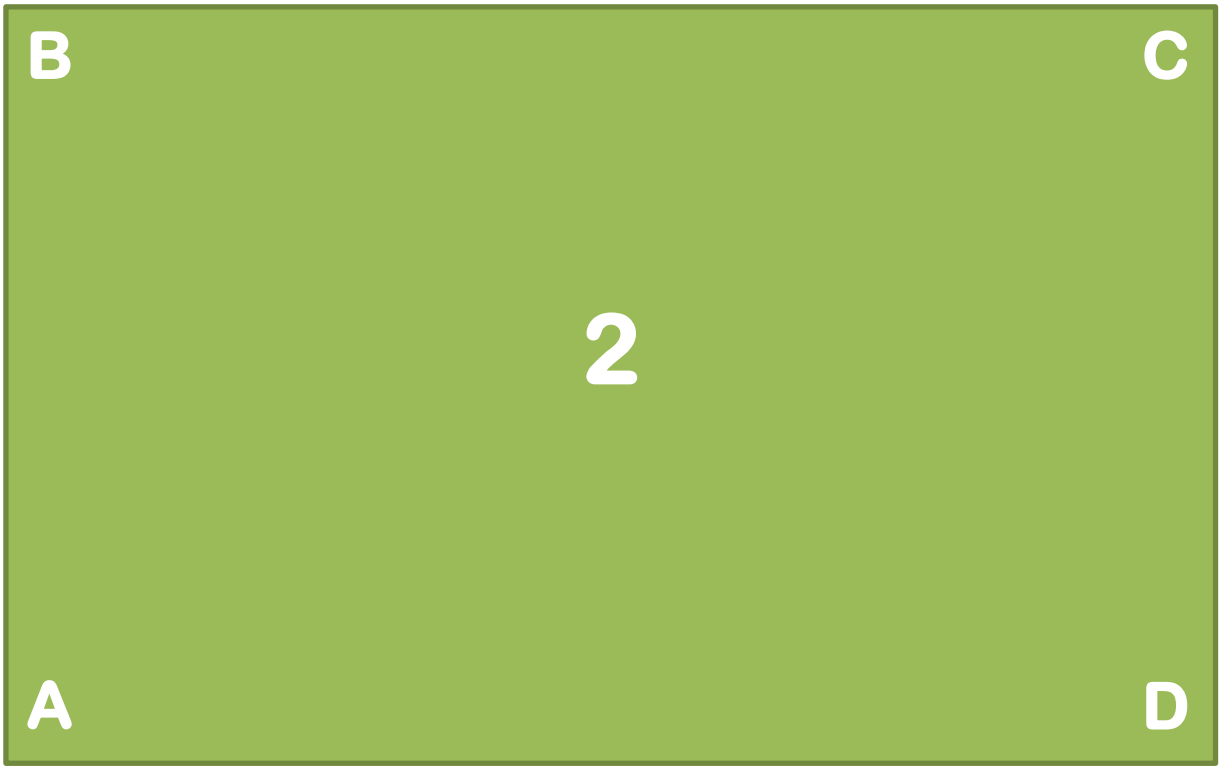
B
C
4
A
D


B
C
1
A
D


B
C
2
A
D


B
C
5
A
D

B

C

5

A

D