NOTES

Logistics

This lesson is intended for students in Grade 6 as an introduction to describing data distributions. Students should ideally be grouped into teams of 4 to promote discussion. A minimum of 24 students is ideally required in order to get enough data for analysis. Extensions to seventh-grade and eighth-grade standards are also provided.

Materials *per student*:

1 – Disposable thermometer
1 – Yellow Post-it note pad sheet
1 – Blue or Pink (depending on male/female) Post-it note pad sheet

per classroom:

1 – Completed number line to post data
1 – Roll blue painter's tape

Time: One to two 45-minute class periods (if including extension)

Objectives/Standards

The objectives of this lesson are to:

- Create and analyze a data set to investigate the question *Is the normal body temperature of humans 98.6°F?* . CCSS.Math.Content.6.SP.A.1 CCSS.Math.Content.6.SP.A.2
- Display the body temperature data using dot plots, histograms, and box plots. CCSS.Math.Content.6.SP.B.4 CCSS.Math.Practice.MP.5
- Discuss the meaning of the variability in the data and brainstorm possible reasons for observed variability in body temperature measurements. CCSS.Math.Content.6.SP.A.3
- (Extension) Draw informal comparative inferences between body temperature data from the male population to those of the female population. CCSS.Math.Content.7.SP.B CCSS.Math.Practice.MP.2
- (Extension) Investigate the relationship between body temperature and heart rate by constructing and interpreting a scatter plot, and informally fit a line to the data if a linear association is indicated. CCSS.Math.Content.8.SP.A.1 CCSS.Math.Content.8.SP.A.2 CCSS.Math.Practice.MP.4

Introduction

This lesson will use a student-generated data set to examine the question, *Is the normal body temperature of humans 98.6°F?*. Data will be collected on the entire group as a whole and will also be collected by gender using different colored post-it notes.

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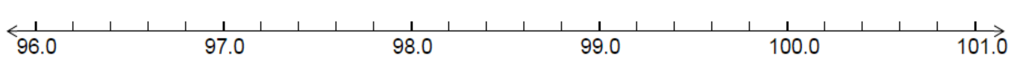
The historical value for mean body temperature of 98.6°F is attributed to a 19th-century German physician, Carl Wunderlich. Dr. Wunderlich conducted his experiments on approximately 25,0000 patients at the University of Leipzig's medical clinic and established the normal range for body temperature of 97.2°F – 99.5°F with a mean of 98.6°F. His description of a range of normal temperatures is often overlooked and the single point value of 98.6°F is identified as the normal body temperature. Wunderlich used axillary, or armpit, measurements and was not overly concerned with precision or calibration of instruments. In the literature, the process used to analyze the data set was never described and the original data set itself was never published. Many scientists now consider 98.2°F to be the true "normal" mean body temperature for humans (Wasserman, et. al. 1992). However, true to Wunderlich's original findings there exists a range of values that are considered normal.

Other contributions Wunderlich made to clinical thermometry have not been as widely publicized, but he was instrumental in pointing out that there appear to be oscillations in body temperature throughout the day. He also believed that women had a slightly higher body temperature than men, and that older individuals also had a lower body temperature.

Other variables that might impact the body temperature of an individual include placement of the thermometer (oral, axillary, tympanic, rectal, or gut (ingested)), circadian rhythms, female menstrual cycles, seasonal (annual) variations, and variations due to physical fitness (Kelly, 2006).

Activity

Students should be grouped into teams of four students. Prior to the activity, use cardstock, sentence strips, or other materials to construct a number line from approximately 96°F to 101°F (scale every 0.2°F). The length of each tick mark along the number line should be approximately the width of a standard-size post-it note sheet. Post the number line along a wall at a height where students have easy access.



Begin this activity by asking students what body temperature is considered normal. The most common answer from students is the historically held norm of 98.6°F. Students should then be asked what they believe it would mean if they had a temperature above this value, or one below this value. Additional questions to ask students prior to data collection include the following:

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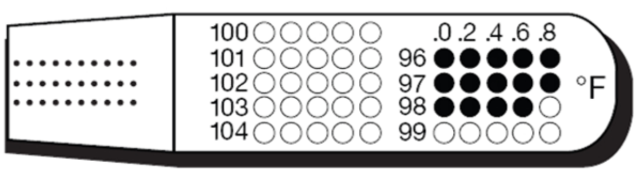
- Do you think other temperatures besides 98.6°F would also indicate a normal body temperature?
- At what temperature would you consider a person to be unhealthy?
- What variables might contribute to an individual's body temperature?

Pass out one disposable thermometer to each student. Ask students to describe the scale that they see on the thermometer. Students should make observations such as the minimum temperature on the thermometer is 96.0°F, the scale is increasing by 0.2°F, and the maximum temperature on the scale is 104.8°F. Then, describe the procedure to determine body temperature using the supplied thermometer below:

- *Remove thermometer from wrapper*
- *Place under tongue as far back as possible and close mouth for 60 seconds*
- *Remove from mouth. Wait about ten seconds for device to lock in accuracy (some blue dots may disappear)*
- *Read temperature indicated by the last blue dot and record using the marker on both Post-its*
- *Throw away wrapper and thermometer*

The last blue dot indicates the correct temperature.



This example reads 98.6°F.

Students will use a yellow, Post-it sticky note sheet to record their body temperature using a marker. Now, allow students to place their sticky note along the number line at a location that corresponds to their measured body temperature. Instruct students that if a sticky note is already placed at their body temperature they are to place their sticky note directly above the one (or ones) that is already displayed. Upon returning to their seats, students should record the class data line plot on their student sheets. At this point, instruct students to record observations about the data set that is displayed on the wall. Encourage students to use some numeric observations about the data set as well and to discuss their observations with the members of their group. Allow time for group discussions, and

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instruct the groups to make a list of all the observations discussed. Begin with one group and have a student representative share their list and record on the board or chart paper. Then, select another group and ask them to share their list. Check off items that are repeated and add new observations that are described. Continue until all groups have reported out and then note the most common observations. Highlight observations related to the following:

- Shape of the overall distribution
- An approximate measure of the center of the distribution
- The range, or variability, in the data
- Interesting or unusual data points within the data set

If no observations lead to the above categories, prompt students to look for additional observations that highlight shape, center, spread, and outliers.

Next, students will construct a histogram of the data set. Using painter's tape, divide the line plot into five "bins" by placing strips of tape just before each whole-number temperature (96, 97, 98, 99, 100, 101). Ask students to describe what values each "bin" or section of the number line would contain. Students should come to an understanding that the first section represents temperatures from 96°F to just below 97°F; the next section from 97°F to just below 98°F, etc. Mathematically, they may represent each section using the notation $96 \leq x < 97$, where x represents the measured body temperature. Rearrange all the post-its in each bin and stack into a single bar. Using painter's tape, place a horizontal piece of tape at the height of the bar with length equal to the length of the bin. Then, remove the sticky notes from that bin. Ask students to describe the meaning of the "bar" that was created. Repeat for the other four bins.

Students will recreate the histogram on their student sheets and will describe again the shape, center, and spread of the distribution. Students will also be asked to compare and contrast the histogram to the line plot. A major goal of this exercise is to allow students to see that individual data points are "lost" when combined into a histogram. However, with larger data sets, it may be impractical to look at a line plot since there might be too much variation to see any meaningful patterns.

Next, arrange the sticky notes along the wall in order from lowest temperature to highest temperature. Ask students to equally divide the sticky notes into four equal groups. Depending upon the size of the data set, students may need to take a sticky note and "rip it in half" to have four equal groups. Again using painter's tape, ask students where you should

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place the tape to create four equal groups. Either place the tape between two sticky notes (indicating that the value of the data will be found by taking an average of the data values on either side of the tape) or down the center of a sticky note (indicating that the particular data value is an “endpoint” for the segments). A sample data set for 10 points 11 points, and 12 points is shown below:

96.2 96.8 97.2 97.6 97.8 98.2 98.4 98.4 98.8 99.4

96.2 96.8 97.2 97.6 97.8 98.2 98.2 98.4 98.4 98.8 99.4

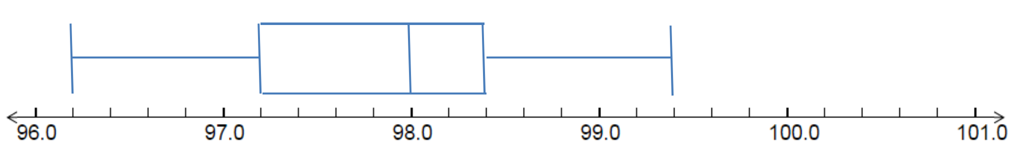
96.2 96.8 97.2 97.6 97.8 98.2 98.2 98.4 98.4 98.8 99 99.4

These values are used to determine the five-number summary to create a boxplot. Each of the lines represents a ***quartile***. Quartile 1, or Q1, means that ¼ or 25% of the data values are below and 75% are above. Quartile 2, or Q2, is also referred to as the median and is the value at which 50% of the data is found above or below. Quartile 3 (Q3) is the value at which 75% of the data is below and 25% is above. The minimum value is the smallest value in the data set, and the maximum value is the largest value in the data set.

For the above sample data sets, the five number summaries are below:

Min	Q1	Median (Q2)	Q3	Max
96.2	97.2	$\frac{97.8+98.2}{2} = 98.0$	98.4	99.4
96.2	97.2	98.2	98.4	99.4
96.2	$\frac{97.2+97.6}{2} = 97.4$	$\frac{98.2+98.2}{2} = 98.2$	$\frac{98.4+98.8}{2} = 98.6$	99.4

These values are used to construct a boxplot such as the one below for the first data set:



Body Temperature °F

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Using the painter's tape, place equal-length strips of tapes at the appropriate values of the five-number summary for the data set on the number line. Then, create the "box" by enclosing the lines between the first and third quartiles (this is known as the interquartile range, or IQR) and draw the "whiskers" by extending a line from the minimum value to the value for Q1 and from Q3 to the maximum.

Following construction, students should use their class data to answer the question:

Is the mean normal body temperature of humans 98.6°F? What does the term "mean" imply? What about the median body temperature?

Students can mark the value of 98.6°F on their graphs and justify their decision based upon their data. Allow student groups to discuss their individual choices, and come up with a consensus for the group. Then, debrief each group again recording responses of "yes" or "no" with relevant justification. Finally, ask students how confident they are with their decision.

Debrief questions:

- What steps might you take to raise the confidence of your decision?
- What variables might play a role in determining an individual's normal body temperature?
- How might you test these variables?

EXTENSION: (Seventh-grade)
To extend this lesson for a seventh-grade classroom, now ask the students to investigate whether or not gender influenced body temperature. Provide a pink sticky note for a female student and a blue sticky note for a male student. Ask students to perform a similar data analysis for these data sets separately. Challenge students to use appropriate graphs to analyze whether a difference exists between the body temperatures of the male students in the class and for the female students in the class. Again, direct students to write a justification for their decision based upon the data and to be prepared to share their choices with the class.

At this point, it is appropriate to discuss with students if their results are representative of the entire population of humans. You may have to direct them towards the idea that this data is only representative of the class, since a random sample of all humans of all ages at various times of the day, etc. was not taken. To get a better idea if the temperature of humans

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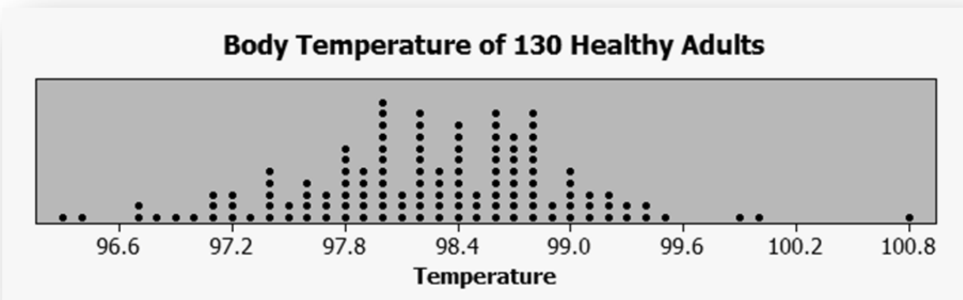
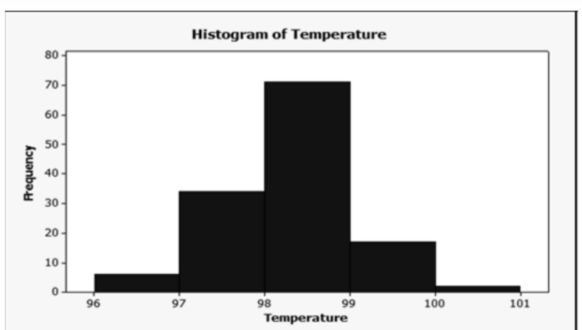
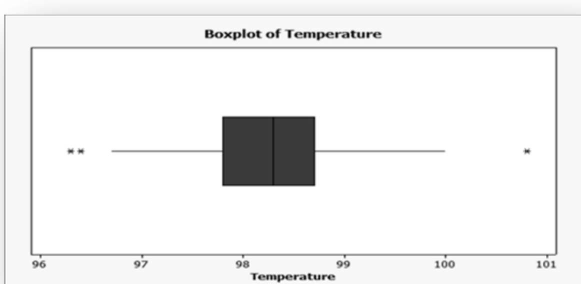
is 98.6°F, students can use a published data set to get more "representative" data.

Navigate to http://www.statcrunch.com/app/index.php?dataid=661807. This data set is entitled "Normal Body Temperature, gender, and Heart Rate" and is a random sample of 130 observations taken directly from data found in a 1992 study (Wasserman, Levine, and Mackowiak) that examined whether the true mean body temperature is 98.6 degrees Fahrenheit. The first column measures body temperature (°F); the second column uses the key 1 = male and 2 = female for gender; and the third column measures heart rate in beats per minute. If the size of the data set is too large, ask students to randomly "sample" the data set for 25 points to use in their analysis. Summary information for the body temperature data set is provided below:



The distribution is bell-shaped with data clustered around the value of 98.3°F. The range of the data is about 4.5°F. One value at 100.8°F seems higher than the rest.





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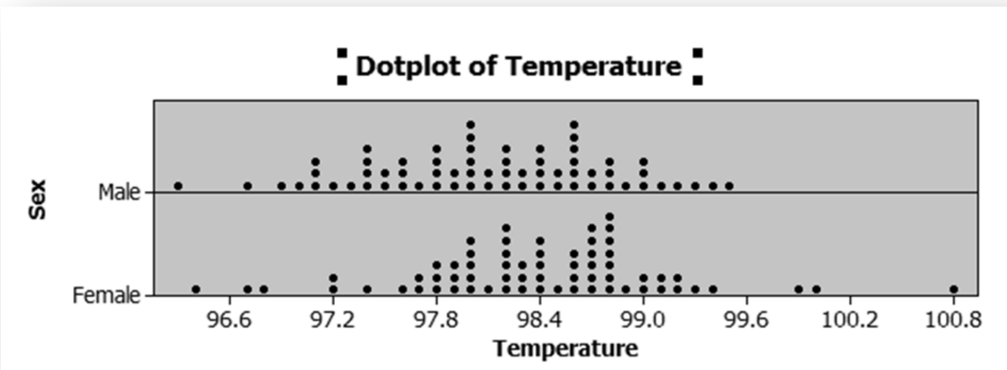
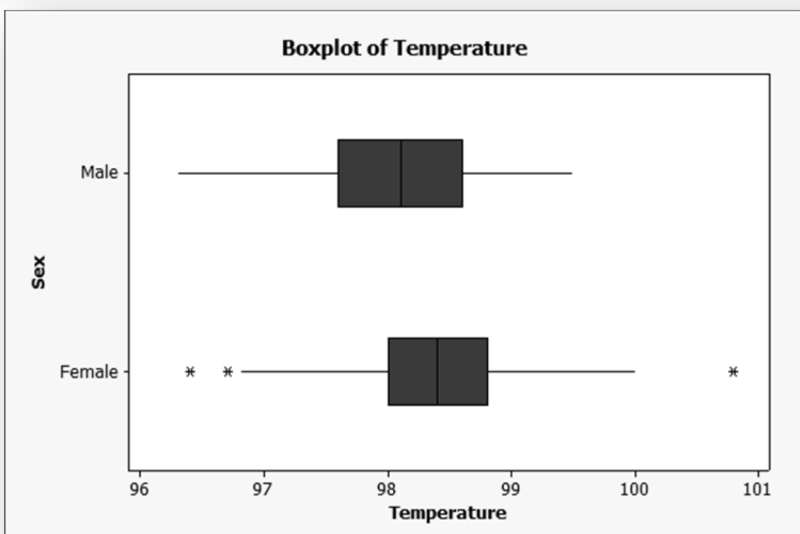
A data set of 130 randomly selected healthy male and female adults' body temperature was taken using a disposable thermometer. The data was taken to investigate the claim that the mean body temperature for the population is 98.6°F. Our data had a median temperature of 98.3°F, lower than the "normal" body temperature, and an interquartile range of 0.9°F which means that 50% of our observations were between Q1 at 97.8°F and Q3 at 98.7°F. The number of observations at or below 98.6°F is 91/130, or about 70%. Two values seemed much lower than the others according to the boxplot, while one value seemed much larger than the others, perhaps meaning that the individual had a fever. Since there were outliers present, the median and IQR were chosen as measures of center and spread.

Students are asked to answer the question of "Is there a significant difference in body temperature between males and females?" The following graphs can be generated to use in this discussion:





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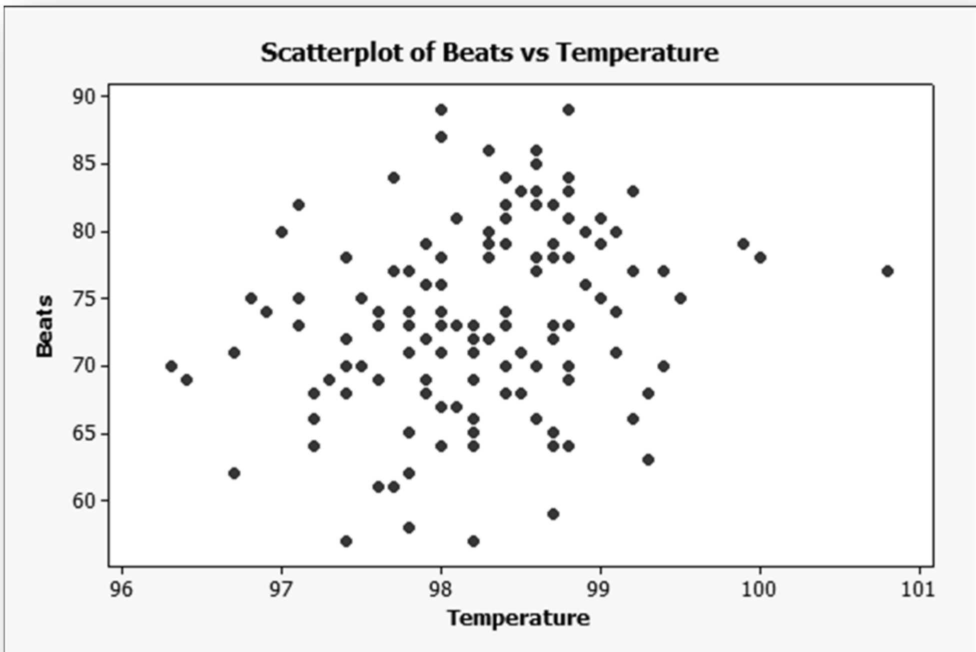
EXTENSION: (Eighth-grade)
To extend this lesson for eighth-graders, students can now use the data set from http://www.statcrunch.com/app/index.php?dataid=661807 to investigate whether there is an association between body temperature and heart rate. Students may form a hypothesis by thinking about a time when they had a fever. Do they recall having a higher heart rate (pulse) while they were ill? Students can use the data to construct a scatter plot and then can informally fit a line to the data. Students should realize that the weakness of the association in the data makes the using the line for predictions subject to error.



Regression line: BEATS = 2.44*TEMP – 166.3

CAUTION: Students can't use the above graph to determine the y-intercept of the line since the scales do not begin at zero. In this problem, the y-intercept has no real-world meaning (a body temperature of zero degrees F has a heart rate of -166.3 beats per minute) However, students can interpret the slope of the line to mean "for every one degree increase in body temperature, the value of heart rate increases by approximately 2.44 beats per minute."

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Conclusion

This lesson is intended to allow students to generate data and use statistical techniques to investigate the question of whether the true mean "normal" body temperature of humans is 98.6°F. Opportunities arise during this lesson to introduce many statistical ideas and generate good discussion of the idea of a central value, variability, outliers, significance, sampling techniques, and experimental design. With NGSS Practice Standards focusing on *Analyzing & Interpreting Data* and *Using Mathematics and Computational Thinking* the connection between statistics and mathematics and science is brought to the forefront. The NGSS standards require key tools from the Common Core Math Standards to be integrated into science instructional materials and assessments. In addition, CC Math Practices MP.2 (Reason abstractly and quantitatively) , MP.4 (Model with mathematics), and MP.5 (Use appropriate tools strategically) are key mathematical practices found integrated into the NGSS standards.

Resources

Kelly, G. (2006). Body Temperature Variability (Part 1): A Review of the History of Body Temperature and its Variability Due to Site Selection, Biological Rhythms, Fitness, and Aging. *Alternative Medicine Review, 11*(4), 278-293.

Mackowiak, P., Wasserman, S., & Levine, M. (1992). A Critical Appraisal of 98.6 Degrees F, the Upper Limit of the Normal Body Temperature, and Other Legacies of Carl Reinhold August Wunderlich. *Journal of the American Medical Association, 268*, 1578-1580.

Shoemaker, A. (1996). What's Normal? - Temperature, Gender, and Heart Rate. *Journal of Statistics Education, 4*(2).

Analyzing Body Temperatures

Dr. Carl Wunderlich, a 19th-century physician, is credited with establishing the "normal" average body temperature of 98.6°F. Wunderlich used armpit temperature measurements on approximately 25,000 patients at the University of Liepzig's medical clinic in Germany. This value for "normal" body temperature is still widely accepted today.

Problem:

Is the average healthy body temperature for humans actually 98.6°F?

Materials:

- One disposable thermometer
- One yellow sticky note sheet
- One pink or blue sticky note sheet
- A dark-colored marker

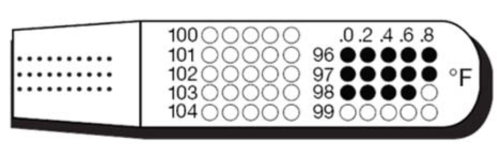
Procedure:

Look at the disposable thermometer you were given and write down any observations on the thermometer below. Be prepared to share your observations with your group.

Use the following procedure to record your body temperature.

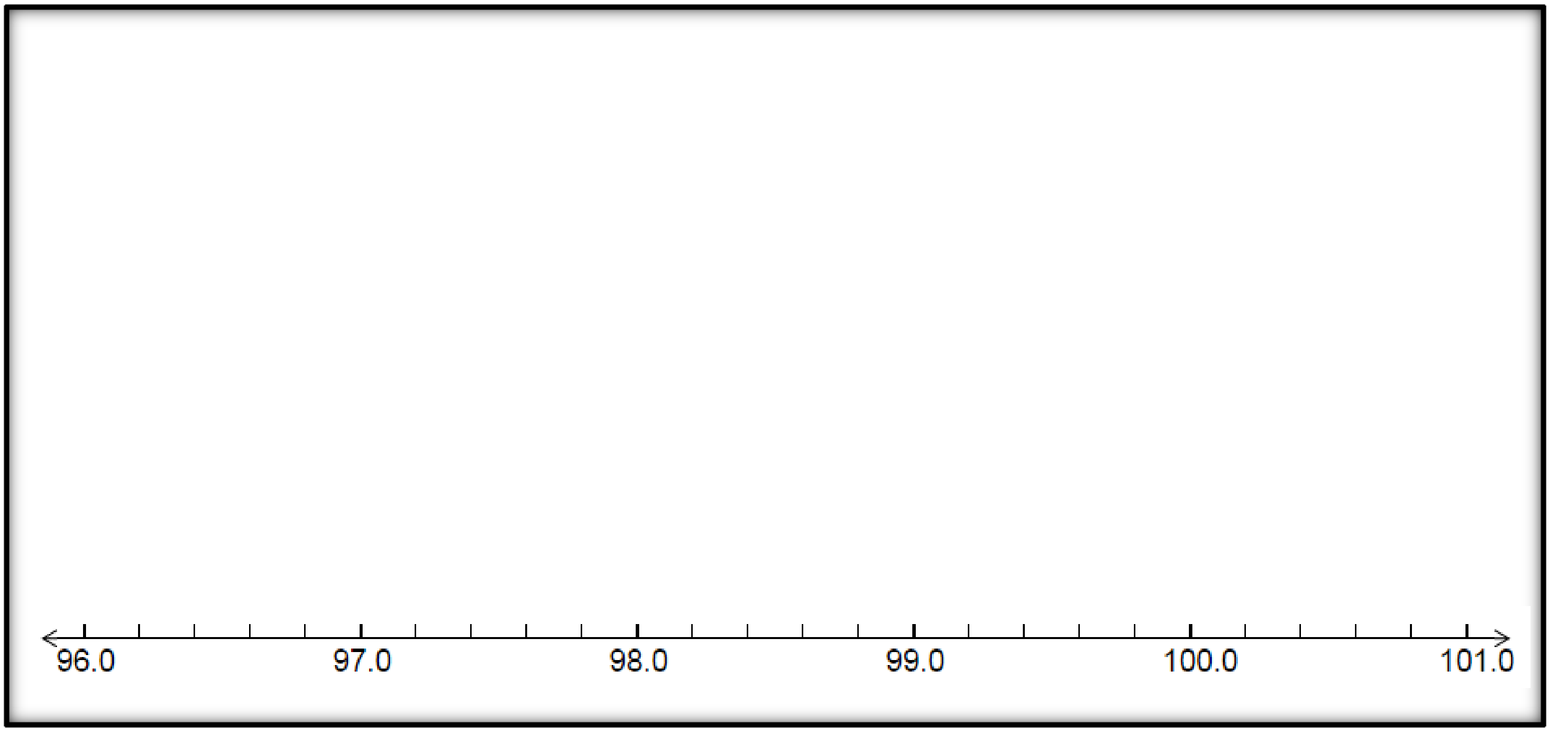
- *Remove thermometer from wrapper*
- *Place under tongue as far back as possible and close mouth for 60 seconds*
- *Remove from mouth. Wait about ten seconds for device to lock in accuracy (some blue dots may disappear)*
- *Read temperature indicated by the last blue dot and record using the marker on both sticky notes*
- *Throw away wrapper and thermometer*

The last blue dot indicates the correct temperature.



This example reads 98.6°F.

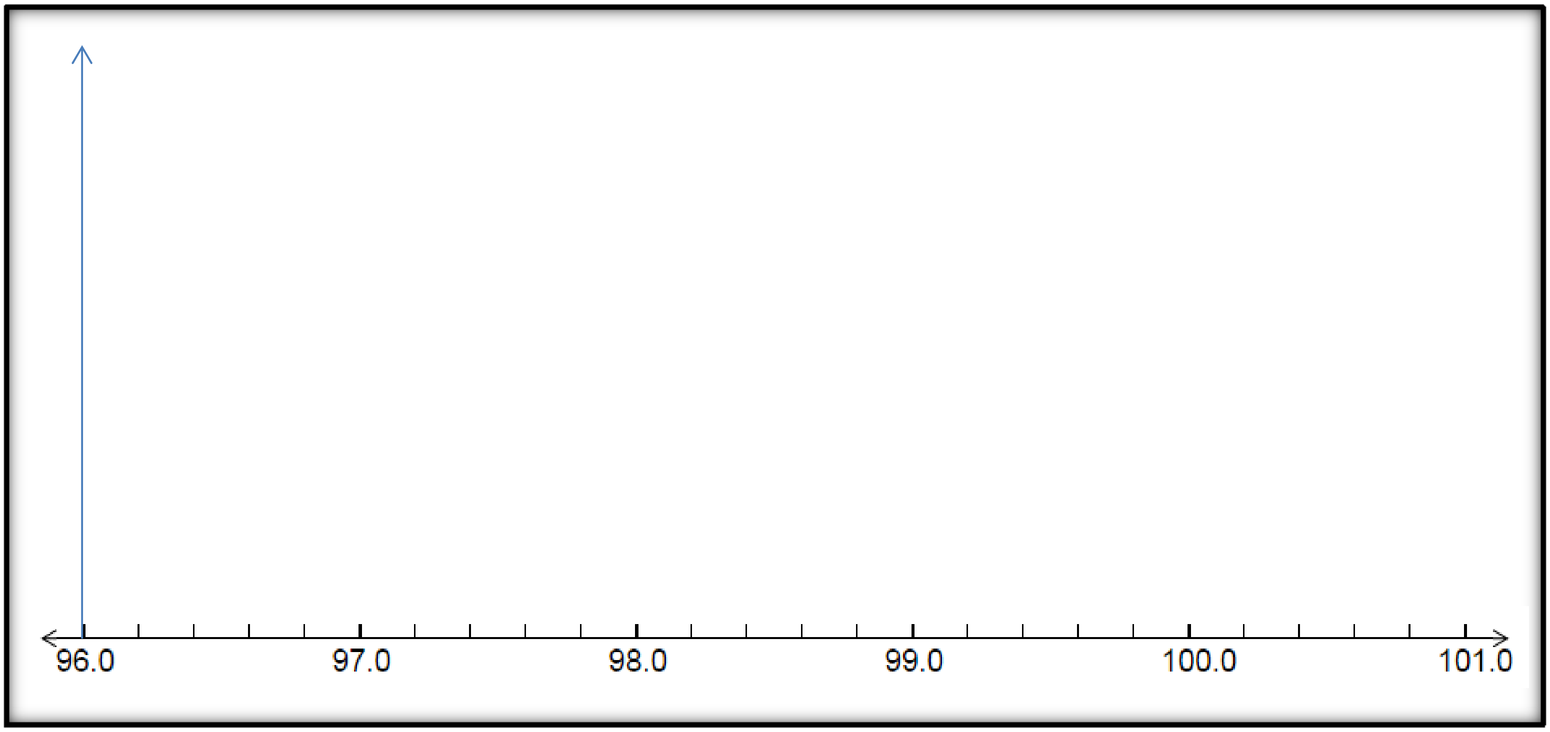
Follow your teacher's instructions to add your yellow sticky note to the data set for the class. When all data points have been added, record the data on the line plot below. What symbol can you use to indicate an individual data point?



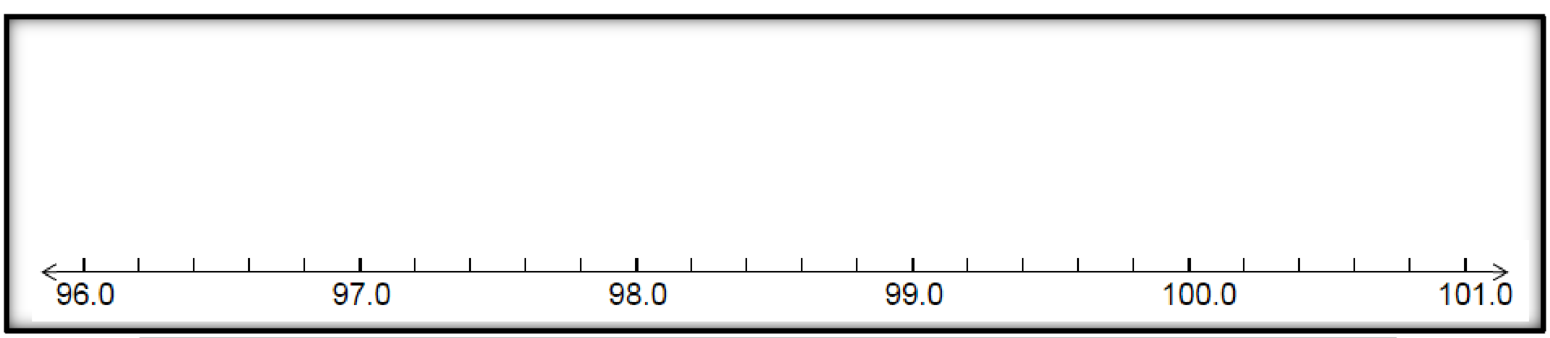
Describe the data set above in as much detail as you can. What are the interesting features of the data set?

Discuss your observations with your group and record additional information that your group discovers about the data set.

Now, create a histogram of the data following your teacher's instructions. How does the histogram compare to the line plot?



Finally, create a boxplot of the data following your teacher's instructions. What might be a possible reason to use a boxplot?



Based upon your data, do you believe that the normal body temperature of humans is 98.6°F? Use your data to justify your decision. You may want to mark this temperature on your graphs to help you make a decision. Once your team has written their responses, share with your group.

How confident are you with your decision?

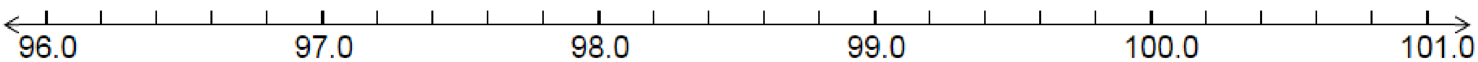
What additional variables might change an individual's "normal" body temperature?

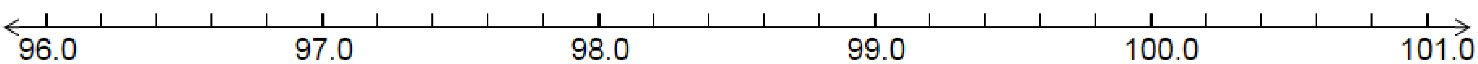
How could you test these variables?

EXTENSION:

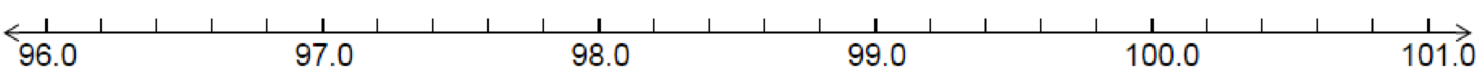
Is there a difference between males and females in normal body temperature?

Using the pink (for female) and blue (for male) sticky notes, create two line plots with which to analyze the above question.





Use the number line below to create another data display.



Why did you choose this display?

Based upon your data, is there a significant difference between males and females in Normal Temperature? Justify your answer.

To investigate a larger data set, navigate to http://www.statcrunch.com/app/index.php?dataid=661807 . What do you think the heading at the top of the columns represent?

Create a display for the data to answer a statistical question of your choice on a sheet of graph paper. Examples of some questions you might use the data set to explore are as follows:

- Is the true mean of the population really 98.6 degrees Fahrenheit?
- What is an acceptable range of “normal” body temperatures?
- At what temperature should we consider someone’s temperature to be abnormal? In other words, when are they sick?
- Is there a significant difference between males and females body temperature in the sample data?
- Is there an association between body temperature and heart rate?

Following your data analysis, write a conclusion to the question you analyzed in the space below.

Introduction to Integers: On Par

Implementing Common Core in Middle School Math: Sixth-grade Lesson

NOTES

Logistics

This lesson is intended for students in Grade 6 as an introductory lesson to the concept of negative numbers. Students should ideally be grouped into teams of 4, and will work independently and as part of a group of four.

Materials per student:

1 – number cube (or other simulation tool)
1 – copy of student pages
1 – PowerPoint Golf presentation

Time: One 45-minute class period

Objectives/Standards

The objectives of this lesson are to:

- Introduce the idea of "par" as the number of golf strokes that you "should" take to move a golf ball from the tee to the hole and simulate using dice or a similar tool golf shots for a hole.
CCSS.Math.Practice.MP5
- Discuss the meaning of "above" par and "below" par. (Understand that positive and negative numbers are used together to describe quantities having opposite directions or values e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.)
CCSS.Math.Content.6.NS.C.5 CCSS.Math.Practice.MP3
- Use integers to mathematically quantify the concept of par. (Use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.)
CCSS.Math.Content.6.NS.C.5 CCSS.Math.Practice.MP4
- Use a number line to graphically represent integers in relationship to zero. CCSS.Math.Content.6.NS.C.6

Introduction

This lesson will introduce the idea that negative numbers can be thought of as directed distances on a number line in relation to zero. Conceptually, this idea will be developed using the game of golf. Students will simulate playing a round of miniature golf using a number cube to simulate the number of shots, or strokes, that it would take them to move the golf ball from the tee to the hole. The "experts" have determined how many strokes it should take for each of the 18 designed holes on the golf course. This number, referred to as "par", will be compared to zero for this simulation. Students will then determine their score by calculating how many strokes above or below the expected value of par they were for each hole of the simulation. Students will then progress to mathematically assigning

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positive values for additional shots and negative values for fewer shots than required. In this way, students can also rationalize that positive does not always indicate "good" and negative "bad"; instead the comparison should be to a standard value such as zero.

Activity

Students should be grouped into teams of four students.

Begin this activity by asking students if any of them have ever played the game of miniature golf. If there are students in the class who have played before, ask them to describe how the game is played. If not, ask if any student has ever seen the actual game of golf played and discuss how the objective is to move the ball from the tee area to the hole in as few shots, or strokes, as possible. Each "Hole" consists of a tee area and a green area for putting. In miniature golf, only putting is allowed and there is usually a maximum of six shots per hole in order to keep play moving.

Pass out the student pages and allow time for students to read the background information. Ask them to discuss with their teammates the answers to the questions posed on the student sheets:

- Can you think of a reason why the maximum number of strokes per hole would be six in miniature golf?
- If you were going to simulate playing 18 holes of miniature golf, what tool or tools might you use to represent the number of strokes it would take to complete each hole?

Elicit student responses to both questions, and list student suggestions for how to simulate playing each hole. Suggestions may include using a six-sided number cube, taking pieces of paper numbered 1 through 6 and placing in a bag or container to draw out individually, using a six-section spinner, using six different colored chips to represent shooting 1, 2, 3, 4, 5, or 6 shots for the hole. As a class, come to a consensus about which tool to use. You may direct students to a particular choice depending upon materials available, or may allow each team to choose how they will model playing each hole.

Now, using the Miniature Golf PowerPoint presentation, display the first hole for students. In the table on their student pages, they should record par for the first hole and then should use their tool to simulate the number of strokes they needed to complete the hole. Next, have students record if they were "above" par for the hole or were "below" par for the hole. Students should reason what they would record if they shot exactly par for

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the hole, but allow students to discuss and arrive at the suggestion to mark either “neither” or “par” for that situation. Students should then calculate how many shots above or below par they were for the hole, and should reason that if they shot par they are zero shots above or below par.

Next, students will calculate the following values for their simulated round of miniature golf:

- *The total number of strokes to complete the course*
- *The number of strokes it should have taken them to complete the course (par)*
- *The number of strokes above or below par they were for the 18 holes*

Students should now discuss with their teams how they might represent mathematically the words “above” and “below” par. Allow time for students to first discuss with their teams, and then have each team report their responses and record on chart paper or the board. Lead students to the idea that a positive value (or +) can be used to represent above par and a negative value (–) can be used to indicate below par.

Now, have students represent the values ***2 strokes above par*** using mathematical notation and using a number line. Do the same for ***2 strokes below par***. Allow time for students to notice that +2 and –2 are the same distance from par, but on opposite sides. Par would best be represented by the value zero since when at par the score is neither above par or below par by any amount. Students move to the right to indicate scoring above par and to the left to indicate scoring below par.

Work through the table with students to help them see how to represent their score for each hole and their total score in relation to par. Allow students to look for patterns to complete the line for Hole 5 and then check their work prior to letting them fill in their table with their simulated data.

Hole	Par	Number of strokes	Above or below par?	Total Par	Total Strokes	Overall score
1	3	4	**+1**	3	4	**+1**
2	3	3	**0**	6	7	**+1**
3	2	4	**+2**	8	11	**+3**
4	4	2	**–2**	12	13	**+1**
5	**3**	1	**–2**	**15**	**14**	**–1**

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If students have correctly filled in their tables, they should notice that the gray box at the end of Hole 18 is the same value as their calculation of the number of strokes above or below par they were for the 18 holes.

Looking at the column ***Above or below par?*** students should discuss with their teammates patterns that they observe. Students may realize that if they group together all of the positive values (those above par) and group together all of the negative values (those below par) they can arrive at the final score by *subtracting* the negative total from the positive total. This is an informal way to introduce adding negative integers as subtracting their opposite, or positive, values.

Using the number line, students should place a mark at ***4 strokes above par*** for Hole 9 and then move to the left 2 spaces to indicate shooting ***2 strokes below par*** for Hole 10 to arrive at +2 as an overall score.

At this point, student teams should now investigate writing a mathematical equation to represent the above situation. Allow time for student teams to come up with equations and then record each team's response on the board. Spend time discussing student responses, especially if they have generated equivalent equations such as $(+4) + (-2) = +2$ or $4 - 2 = 2$.

Finally, allow students to solve the equation $(+2) + (-3) = -1$ and interpret this to mean "if a current score was two shots above par, and the next hole was three shots below par, then the overall score would now be at one shot below par."

Debrief questions:

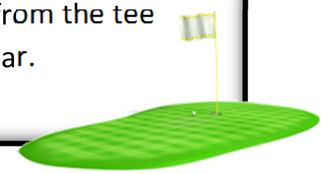
- What do positive and negative numbers mean in relationship to zero?
- How can you combine (add) two positive numbers? Two negative numbers? One positive and one negative number?
- What other real-world situations use positive and negative numbers?

Conclusion

This lesson is intended to allow students to investigate positive and negative numbers within the context of the game of golf. It is desired that students can then informally address the "rules" for combining integers by reasoning that if they shoot two scores "above" par then they will be above par; two scores below par will still be below par; and one above and one below depends upon the "higher" value above or below par.

Introduction to Integers: On Par

The game of golf is played by using a club to strike a small round ball from an area called the "tee" into a round hole a distance away from the tee. Each time the ball is struck by the club, a "stroke" is recorded on the player's scorecard. Experts have determined how many strokes it should take a player to hit the ball from the tee to the hole. This number is referred to as **par**. So, if par for a hole is 3, that means it should take you three strokes to hit the ball from the tee to the hole. If you take four shots, you are ***above*** par; two shots would be ***below*** par. Your goal is to complete the course in the fewest amount of strokes.



In miniature golf, scoring is similar to the game of golf played on a large course. Miniature golf is played by "putting" the ball from the tee area into the hole. Most miniature golf courses only allow a maximum of six shots, or strokes, per hole, and you must take at least one shot per hole. Can you think of a reason why the maximum number of strokes per hole would be six?

If you were going to **simulate** playing 18 holes of miniature golf, what tool or tools might you use to represent the number of strokes it would take to complete each hole?

Using the tool decided upon by the class, simulate a round of miniature golf by completing the scorecard below. Your teacher will tell you the par for each hole.

Hole	Par	Number of strokes	Above or below par?	By how many strokes?	Hole	Par	Number of strokes	Above or Below?	By how many strokes?
1					*10*				
2					*11*				
3					*12*				
4					*13*				
5					*14*				
6					*15*				
7					*16*				
8					*17*				
9					*18*				

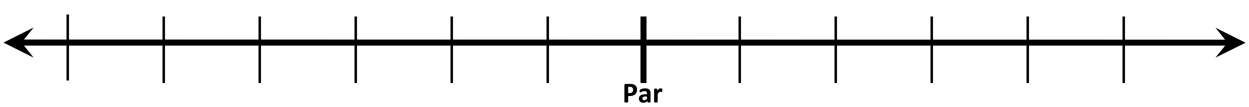
What were the **total** number of strokes it took you to complete the miniature golf course?

How many strokes **should** you have taken?

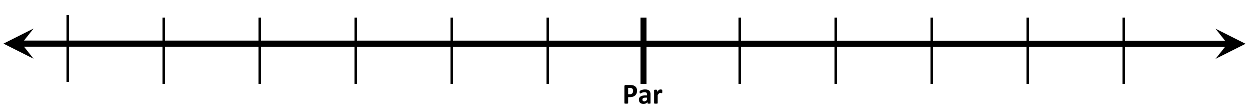
How many strokes **above** or **below** par were you for the 18 holes?

With your teammates, discuss how you might represent the words “above” and “below” using mathematical symbols. Be prepared to discuss your method with the rest of the class. Write an example of how you would show ***2 strokes above par*** and ***2 strokes below par*** using your method in the space below:

Using the method discussed by the class, show what ***2 strokes above par*** would look like on the number line below:



What would ***2 strokes below par*** look like on the number line below?



What number would you place where the word “Par” is on the number line? ______
Why?

What direction are you moving for holes “above” par? ________ Below par? _________

Now, using the data you recorded from your simulation, use the notation of (+) to represent the number of strokes above par and (–) to represent the number of strokes below par for each of your holes. Then, try to keep a running total of your overall score relative to par. An example is shown below for the first few holes:

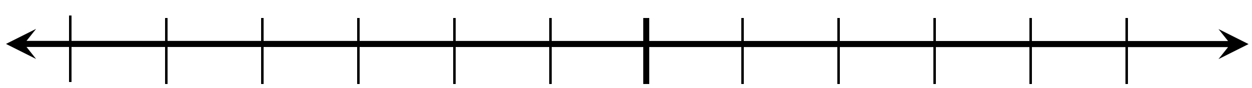
Hole	Par	Number of strokes	Above or below par?	Total Par	Total Strokes	Overall score
1	3	4	**+1**	3	4	**+1**
2	3	3	**0**	6	7	**+1**
3	2	4	**+2**	8	11	**+3**
4	4	2	**–2**	12	13	**+1**
5	**3**	1				

Hole	Par	Number of strokes	Above or below par?	Total Par	Total Strokes	Overall score
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						

Looking at your overall score relative to par at the end of Hole 18 (the box that is shaded gray), compare that number to your first calculation of how many strokes **above** or **below** par you were for the 18 holes.

Now look at the column ***Above or below par?*** How can you use the numbers in this column to calculate your overall score at the end of the 18 holes? Discuss with your teammates and record your strategy below.

Using the number line below, show how you can determine what your overall score is if you are currently ***4 strokes above par*** at Hole 9 and you score ***2 strokes below par*** on Hole 10.



Write a mathematical equation that shows your overall score at the end of Hole 10.

Share your equation with your teammates. How are your equations the same? How are they different?

Solve the equation $(+2) + (-3) = ?$ Then, write a golf situation that would use this equation.

Deriving the Circumference Formula

Implementing Common Core in Middle School Math: Seventh-grade Lesson

NOTES

Logistics

This lesson is intended for students in Grade 7 as an introductory lesson to derive the formula for circumference and investigate the relationship between circumference and area of a circle. Students should ideally be grouped into teams of four and will work independently and as part of a team.

Materials per student:

1 – flattened paper plate
1 – ruler
1 – meter of kite string or yarn
1 – pair scissors
1 – sheet cm grid paper
1 – copy of student pages

Materials per team:

1 – can tennis balls
4 – additional circular objects to measure

Time: One to two 45-minute class periods if doing extension

Objectives/Standards

The objectives of this lesson are to:

- Measure the circumference (distance around) and diameter (distance across) for various circular objects. CCSS.Math.Practice.MP5
- Determine that a constant ratio exists between the circumference of a particular circle and the corresponding diameter of that circle. CCSS.Math.Content.7.RP.A.2.A CCSS.Math.Practice.MP2
- Use the idea of circumference to predict the distance traveled by counting the number of revolutions of a bicycle tire. CCSS.Math.Content.7.G.B.4 CCSS.Math.Practice.MP1
- Informally relate the circumference of a circle to its area CCSS.Math.Content.7.G.B.4 CCSS.Math.Practice.MP7

Introduction

In this lesson, students will informally derive formulas to measure the circumference and area of a circle. Students will then apply their understanding of circumference to determine the distance traveled by a rotating bicycle tire.

Activity – Part 1 *Deriving a formula for the circumference of a circle*

In the first part of this activity, students will be working in teams of four to discover the relationship between the diameter of a circle and its circumference.

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Provide student teams with circular objects of various sizes, pieces of string or yarn large enough to fit around the largest object, and a ruler. Direct students to measure the length around each of the circular objects to the nearest mm, and record the information in the table on the student pages. Next, students will measure the diameter of each circular object and will also record that information in the table. Remind students that a diameter of a circle must pass through the center of the circle.

Students will then calculate the ratios of the circumference of each circle to its corresponding diameter. Allow each team to come to the board and list their ratios for each of the objects they measured. Students should recognize that the ratio is approximately 3 (close to the value of pi, π) for any circular object.

Next, provide each team with a can of three tennis balls (preferably unopened). Challenge each team to cut a piece of string or yarn that would exactly fit around the circumference of one tennis ball. Students can rationalize that the height of the three tennis balls is approximately equal to the circumference of the ball, or can try to measure a diameter of one ball and use their ratio to calculate the length of the piece of string. When each team has cut their piece of string, allow students to open the can and remove one ball to test their calculation.

Pose the following questions to students:

- Was the piece of string your team cut too long, too short, or exactly right? [Answers will depend upon student calculations]
- How could you determine the exact amount of kite string to cut? [We could use a formula and measure the diameter precisely]
- Using a calculator, the value of pi that your calculator returned is about 3.141592653589. How does this value relate to the ratios that were calculated between circumference and diameter? [The value of the constant, pi, is close to the ratios we experimentally determined by measuring the circumference and diameter of the various circles.
- What formula did your team write for the circumference of any circle? [Circumference = $\pi \times$ Diameter]

Next, student teams will work to solve the bicycle problem posed to them on the student sheets. Allow teams plenty of time to work through the problem. A sample solution is shown on the following page.

NOTES

$$Distance = 400\ rev \times \frac{\pi\ \times 330\ mm\ \times 2}{1\ rev} \times \frac{1\ m}{1000mm} \approx 829.4\ m$$

If 1 mile = 1609.34 meters, then the distance traveled is about 0.5 miles.

Activity – EXTENSION Part 2 *Deriving a formula for the area of a circle*

Students will continue working in teams of four for this extension to the first activity.

Provide each student with a flattened paper plate (the inexpensive paper plates work the best – no foam plates), a pair of scissors, a ruler, and a piece of yarn or string.

Students will begin this part of the activity by first measuring the circumference of the paper plate. They may also measure the diameter and use their formula to calculate the circumference to reinforce the previous lesson.

Next, provide teams with sheets of cm grid paper and have students trace the paper plate onto the grid paper. Count the squares, estimating partial squares, to arrive at an estimate for the area of the paper plate.

Now have students follow the procedure on the student pages to cut increasingly smaller equivalent sectors of the circle to use the pieces to create a parallelogram. Sixteen equal pieces is about the maximum students can cut before it becomes difficult to construct the parallelogram.

Ask student teams to recall the formula for area of a parallelogram (***A = b×h)***. If students are having difficulty with this formula, ask them to recall the formula for area of a rectangle.

Allow time for student teams to relate the formula for area of a parallelogram to the circumference of the circle they used to create the parallelogram. Students should recognize that ½ the curved edges (1/2 the circumference) forms the bottom base of the parallelogram with the other ½ forming the top side of the parallelogram. Students should also recognize that ½ of the diameter (the radius) forms the sides of the parallelogram.

Substituting into the formula:

$$Area = \left(\frac{1}{2}Circumference\right) \times \left(\frac{1}{2}Diameter\right)$$

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Mathematically simplifying this relationship gives us the common formula for area of a circle:

$$A = \frac{C}{2} \times \frac{d}{2} = \frac{\pi d}{2} \times \frac{d}{2}$$

$$If\ r = \frac{d}{2}, \text{ then } A = \pi r \times r \text{ or } A = \pi r^2$$

To calculate the area of the circle, students can take ½ the circumference and multiply by ½ the diameter. They should then compare this result to the estimated area they determined using the cm grid paper.

Debrief questions:

- How does the circumference of a circle relate to the diameter of a circle?
- How can I use the circumference of a circle to measure distance traveled for a rotating object?
- How does the area of a circle relate to the circumference of a circle?

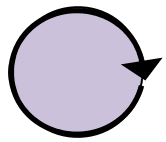
Conclusion

This lesson allows students to discover the relationships between the diameter, circumference, and area of a circle. The relationship between circumference and distance traveled by a rotating object is also explored with the goal of students relating circumference to a measurement of distance.

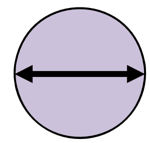
Deriving the Circumference Formula

In this activity, you will be working with a team of four students. In your team you will be measuring different-sized circles. You will be recording two different measurements:

Circumference
(distance around the circle)



Diameter
(distance across the circle through the center)



Record your measurements to the nearest millimeter (tenth of a centimeter).

Description of Circle	$\boldsymbol{Circumference}$	$\boldsymbol{Diameter}$	Ratio $\frac{\boldsymbol{Circumference}}{\boldsymbol{Diameter}}$

What observations can you make about the ratio between the circumference and diameter?

Deriving the Circumference Formula
Student Pages

Your team will be given a can of tennis balls. Without opening the can of tennis balls, cut a piece of kite string that will fit exactly around the circumference of one of the tennis balls. Describe your procedure below, including any calculations:

Remove one of the tennis balls from the can and test the accuracy of your prediction.
Was the piece of string you cut too long, too short, or exactly right?

How could we determine the exact amount of kite string to cut?

Using a calculator, press the button for the value of pi (π) and record the value presented:

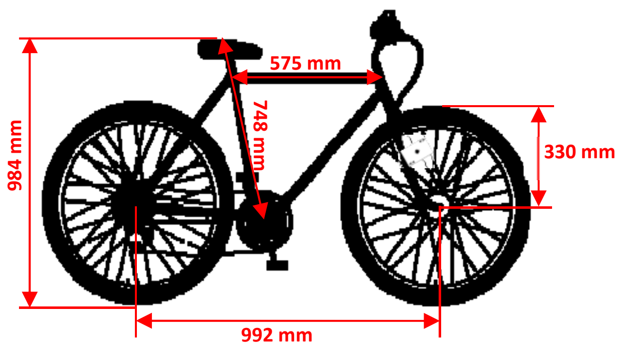
How does this value relate to the ratios that were calculated between circumference and diameter?

Write a formula for the circumference of any circle:

Your team has been given the following problem to solve:

You have placed a playing card  between the spokes of your new bicycle to make a cool clicking noise while you ride to your friend's house. During your ride, you count that it took 400 clicks to reach your friend's house. How far did you ride?

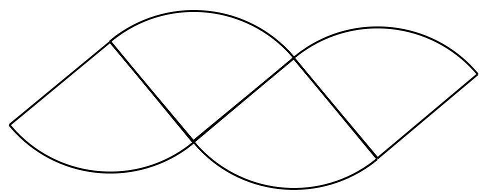
Your bicycle has the following dimensions:



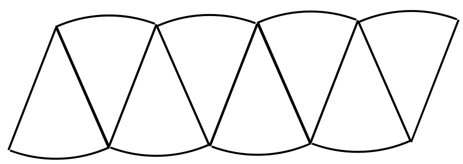
Compare your team's answer to those of other teams. How are they alike? How are they different?

Extension: Relating Circumference of a Circle to Area

1. Measure the circumference of a flattened paper plate to the nearest mm. ___________
2. Trace the paper plate onto cm grid paper and estimate the area of the paper plate.
3. Fold the paper plate into four pieces and cut into four sections. Arrange the sections to form a parallelogram as shown below:



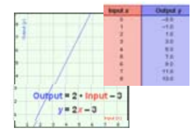
4. Now, cut each of the pieces in half to make eight sections and again arrange to form a parallelogram.



5. Compare the parallelograms from Steps 3 and 4.
6. Once again, cut each piece in half to make 16 sections and arrange to form a parallelogram. What is the measure of the height of the parallelogram?

   What is the measure of the base of the parallelogram?

7. What is the formula for area of a parallelogram?________ Use this formula to write another formula to represent the area of a circle in terms of the circumference.
8. Use your formula to calculate the area of the circle and compare to your estimate in Step 2.



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Logistics

This lesson is intended for students in Grade 8 to extend their working knowledge of multiple representations of linear functions. Students will be working in teams of two during this activity.

Materials per student:

1 – set of Matching Cards (Four different colors)
1 – copy of student pages

Time: One 45-minute class period

Objectives/Standards

The objectives of this lesson are to:

- Construct a function to model a linear relationship between two quantities given a verbal description. CCSS.Math.Content.8.F.B.4 CCSS.Math.Practice.MP4
- Justify selection of cards to model a given problem situation. CCSS.Math.Practice.MP3
- Determine the rate of change and initial value of the function from a description, a table, or a graph. CCSS.Math.Content.8.F.B.4
- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). CCSS.Math.Content.8.F.A.2
- Evaluate functions for a given input or output value. CCSS.Math.Content.8.F.A

Introduction

In eighth grade, students are formally introduced to the concept of functions where one quantity determines the other. They learn to translate multiple representations of a function, even when shown only partial information. Also, students are introduced to the idea of slope as a rate of change between two quantities, and can interpret the meaning of the slope and y-intercept within the context of a particular problem. This lesson would be most appropriate as a formative assessment to gauge student understanding of these concepts.

Activity

Part 1: Matching Verbal Descriptions using Multiple Representations

Students should be grouped into teams of two students.

Begin this activity by passing out the first student page containing the data recording table. Allow students time to read the instructions, and then

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pass out a set of Matching Cards for students. It is desired to copy each sheet of matching cards (Problem Cards, Algebraic Function Cards, Graph Cards, and Table Cards) on a different color or paper. Students should be provided scissors if they are going to cut the cards out for this activity.

Next, allow student teams time to sort into six sets one matching Problem Card, Algebraic Function Card, Graph Card, and Table Card per set. Students do not need to go in order of the Problem Cards, and can start with any problem. To aid in assessing student progress while the teams are working, the following chart may be used:

Problem Card	Algebraic Function Card	Graph Card	Table Card
1.	F (or A)	b (or e)	☺ (or ▲)
2.	E	a	✱
3.	A (or F)	e (or b)	▲ (or ☺)
4.	D	f	✓
5.	C	d	💧
6.	B	c	★

If students are having difficulty sorting the cards, you may use the following prompts to assist.

- *Can you tell me what the problem is asking you to model in your own words?*
- *Does this problem remind you of others you have solved?*
- *What part of the problem might represent something that is constant, and what part of the problem might represent something that is changing?*
- *What clues do linear functions give you when looking at their graphs?*
- *What are good points to look for on graphs and in tables?*
- *What might the x and the y refer to in the problem?*

Emphasize to students the importance of writing a justification for each set of cards they combine to represent one problem. Choose one team to list a set of cards that they have matched together, beginning with the Problem Card number. Allow the team to state why they matched these together.

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Then, ask the other teams if they have any other combination of cards for that particular Problem Card. List all student responses and then allow the students to debate which cards they believe match the Problem Card chosen. At this point, do not state whether students are right or wrong, simply ask for a consensus set of matching cards before moving on to the next Problem Card. Continue until all six sets have been debated.

Then, announce how many set are "correct" (2/6, 4/6, etc.). If there are errors, do not indicate which set contains an error, but let students decide which sets to check for errors. Students can use the table to check ordered pairs on the graph, and then can use the input/output pairs to check the validity of the function. The difficult part is making sure that the representations model the chosen problem. If that is the source of the error, you may direct students to check their models once again.

Debrief questions:

- Were some problems easier to model than other problems? Why?
- Were you stuck at any point? How did you get "unstuck"?
- Did some problems have more than one solution? Why or why not?
- What was tricky about some of the Graph Cards?

Part 2: Comparing Multiple Representations

Students should be grouped into teams of two students.

In this part of the activity, students will use the cards from Part 1 to answer a series of questions. Pass out the next two Student Pages and allow students time to read the instructions. Several of the questions are open-ended, and students may have different answers depending upon their interpretation of the question. Again, emphasize the importance of stating WHY a particular card or cards were chosen to illustrate a particular problem.

Question 7 uses the cards to evaluate situations based upon the models selected for a particular Problem Card. This question can be eliminated if time constraints are present, or used as a review in a subsequent class session.

Answer Key for Question 7:

Problem 1: 1) 5 2) 10/4 or 2.5 pieces per day 3) It is possible if the candy can be cut in half.

Problem 2: 1) $34

NOTES

Problem 3: 1) 5 pairs of jeans 2) 10 t-shirts

Problem 4: 1) $122 2) 48 books

Problem5: 1) $2

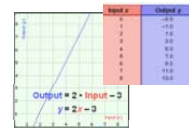
Problem 6: 1) $24
2) 3books for $6/month ; exactly 12 books in 4 months
3) No; 16 is not a multiple of 3 or 5
4) Choice 1; I will have at least 16 comic books after 6 months with Choice 1 ($36 fee); but I would have to buy 4 months of Choice 2 ($40 fee) to have at least 16 books

Debrief questions:

- Which Table Cards did you select with the same rate of change? Why?
- Which Algebraic Function Cards did you select with the same rate of change? Why?
- Why are most of the graphs shown using only the first quadrant?
- Which cards show a positive rate of change? How do you know? How could you tell a positive rate of change on the Algebraic Function Cards?
- Which cards show a negative rate of change? How do you know? How could you tell a negative rate of change on the Table Cards?
- How can you tell if one graph shows a greater rate of change than another graph? How could you check your idea?

Conclusion

This lesson is intended to follow formal instruction related to the slope-intercept form of a linear function. A key strategy here is to allow students to work collaboratively and talk mathematically to each other to accomplish a task. The debriefing of student findings is essential to counter any misconceptions students may have and to clarify their thinking on a particular concept. Further instruction can be developed to reteach these concepts to students if necessary, or to move towards data collection and informally fitting a line to a set of data that is linear.

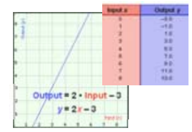


Matching Multiple Functional Representations

In this activity, you and a partner will be sorting a series of cards. Each set of cards contains one problem card, one algebraic function card, one graph card, and one table card. For each set of cards that you sort, write the code (number, letter, or symbol) from the card in the table below. Then, write a justification for why you placed these cards together in the last box of the table. Good luck!

Problem Card	Algebraic Function Card	Graph Card	Table Card	Justification

Be prepared to discuss how your team decided to sort the cards with your classmates.



Comparing Multiple Functional Representations

In this part of the activity, you and your partner will now use the cards to answer the questions below. There may be more than one to answer a question. Please be sure to write WHY you selected the cards you did to answer a particular question.

1. Choose two **Table Cards** that show the same rate of change: _____ and _____
   Why did you select these cards?

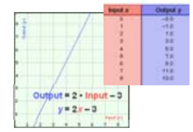
2. Choose two **Algebraic Function Cards** that show the same rate of change: _____ and _____
   Why did you select these cards?

3. For the **Graph Cards**, why do you think only the first quadrant of each graph is shown?

4. Find all of the **Graph Cards** which show a POSITIVE rate of change:
   ____
   Why did you select these cards?

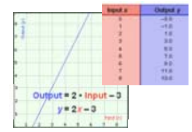
5. Find all of the **Graph Cards** which show a NEGATIVE rate of change:
   ____
   Why did you select these cards?

6. Compare **Graph Card d.** and **Table Card ✱**. Which has the greater rate of change? _____
   Why?

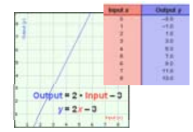


7. Use your cards to help you answer the following questions:

Problem Card 1 1) How many days would it take you to eat all of your candy? 2) If you wanted to eat all of the candy in 4 days, what would the rate of change of pieces of candy have to be per day? 3) Is that possible? Why or why not?	**Problem Card 2** 1) How much would you have to pay the online gaming service if you downloaded 12 games?
Problem Card 3 1) What is the maximum number of pairs of jeans you can buy? 2) What is the maximum number of t-shirts you can buy?	**Problem Card 4** 1) How much does the book club cost for 1 year? 2) How many total books would you receive in that year?
Problem Card 5 1) How much more would your bill be if you used 500 minutes in one month compared to 400 minutes in that same month?	**Problem Card 6** 1) How much would exactly 12 Comic books cost? 2) Which plan would you use to buy exactly 12 comic books? _____ Why? 3) Could you purchase exactly 16 Comic Books with either of these plans?_____ Why or why not? 4) If you wanted 16 Comic Books, what would be the best plan choice?



1. Problem Card Your favorite candy is on sale at the local grocery store. You decide to buy 10 pieces of candy but don't want to eat it all at once. Instead, you decide to eat only 2 pieces per day. How can you model how much candy you have each day using a function, table and graph?	**2. Problem Card** You are interested in trying out an online gaming service. The sign-up fee is \$10, and you can then download video games for \$2 each. How can you model the total cost of the online service using a function, table and graph?
3. Problem Card You have earned \$100 over the summer mowing lawns. You want to buy jeans and t-shirts with the money, but don't know how much of each to buy. A pair of jeans cost \$20 and a t-shirt costs \$10. How can you model the combinations of jeans and t-shirts you can purchase using a function, table and graph?	**4. Problem Card** To join a book club, you are charged an initial fee of \$2 and then are charged \$10 per month for which you receive 4 new books per month. How can you model the total cost of the book club using a function, table and graph?
5. Problem Card A cell phone company charges \$10 per month for the service plus 2¢ per minute that the phone is in use. How can you model the total cost of your monthly phone bill based on minutes the phone was in use during the month using a function, table and graph?	**6. Problem Card** You are purchasing a subscription from a *Comic Book of the Month* club. You have a choice of (1) purchasing 3 comic books per month for a cost of \$6 or (2) purchasing 5 comic books per month for a cost of \$10. Choose **one option** and model the cost **per comic book** using a function, table, and graph.



A. Algebraic Function Card

$$100 = 20x + 10y$$

B. Algebraic Function Card

$$y = 2x$$

C. Algebraic Function Card

$$y = 0.02x + 10$$

D. Algebraic Function Card

$$y = 2 + 10x$$

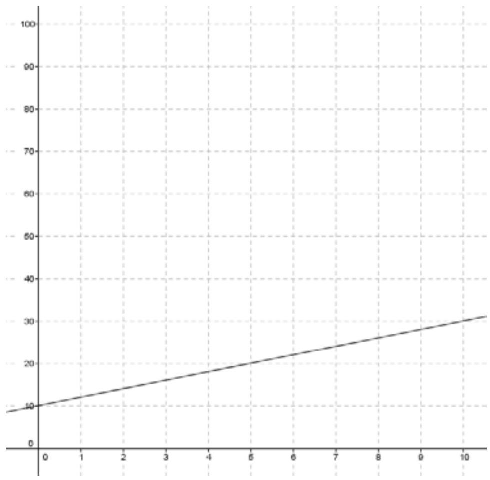
E. Algebraic Function Card

$$y = 2x + 10$$

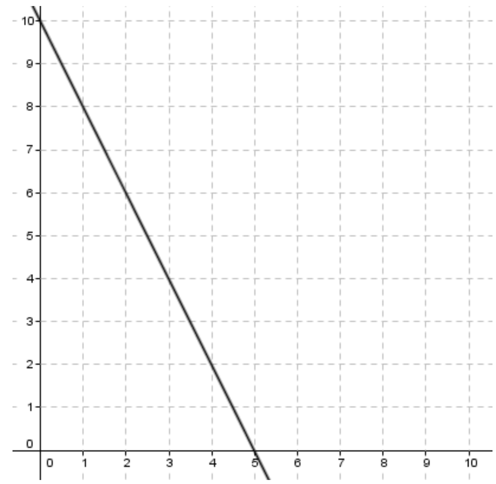
F. Algebraic Function Card

$$y = 10 - 2x$$

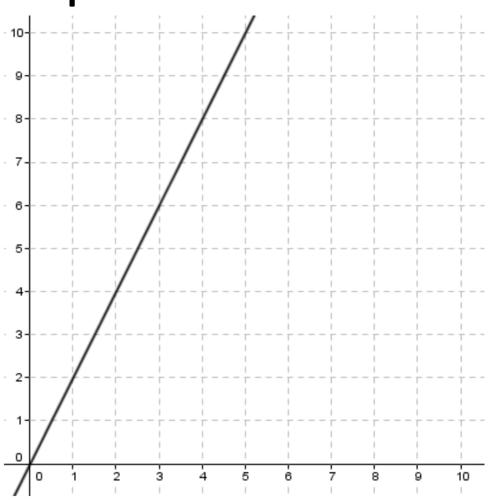
a. Graph Card



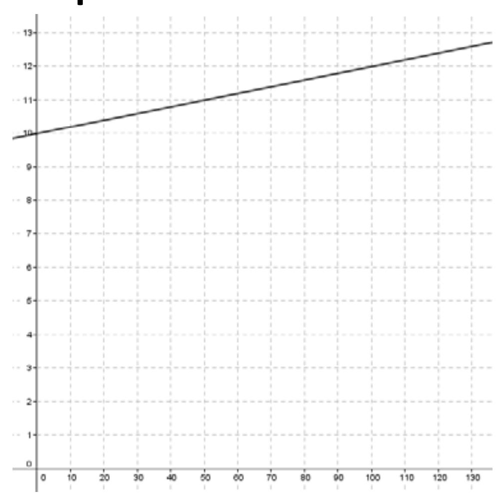
b. Graph Card



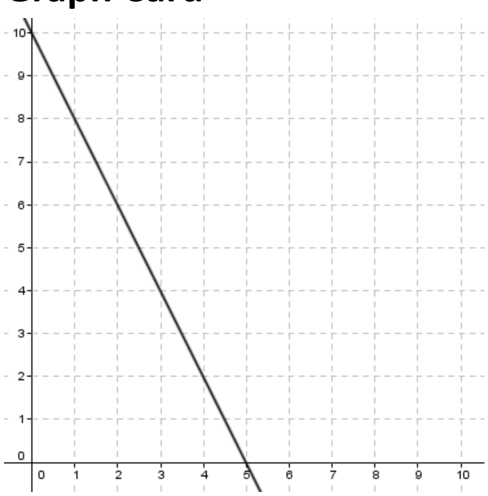
c. Graph Card



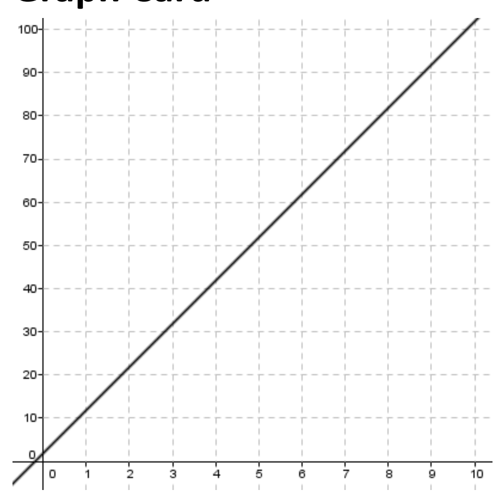
d. Graph Card



e. Graph Card



f. Graph Card



✓ Table Card

x	y
2	22
4	42
6	62
8	82
10	102

★ Table Card

x	y
1	2
3	6
5	10
7	14
9	18

☺ Table Card

x	y
1	8
2	6
3	4
4	2
5	0

▲ Table Card

x	y
0	10
1	8
2	6
3	4
4	2

🌢 Table Card

x	y
0	10
20	10.4
50	11
80	11.6
100	12

✱ Table Card

x	y
0	10
2	14
5	20
8	26
10	30