Snowflakes and Chaos

Objective:
Explore how the principles of sensitive dependence on initial conditions and iteration allow a simple process to give rise to complexity.

Estimated Time: 60 minutes

Materials:
for each student:
- Student Handout
- 1 - Sheet of triangular graph paper

for each group of 2 or 3:
- 1 - Spinner or die (10-sided)
- 3+ colored pencils

for teacher:
- Snowflakes and Chaos Slideshow

Suggested Inquiry Approach:

Begin by asking students to describe what a snowflake looks like (using only words). This is tougher than you'd think even though everyone recognizes a snow flake when they see one.

Now begin the slideshow *Snowflakes and Chaos.*

First, students will study the image silently. Then, help them form groups of two or three to share and compare their ideas. Finally, open a class discussion and invite each group to contribute their observations. Encourage them to defend their ideas by citing evidence from the image.

The next three slides describe the basic process by which snowflakes form. The remainder of the slide show explains the steps to simulating the process of snowflake formation.

Give each student a sheet of triangular graph paper and the Student Handout. Give a spinner or 10-sided die and some colored pencils to each group. Assist as students go through three iterations of the process to form a snowflake. Emphasize the following:

- ❄ Their color selection is not important. Colors just help to tell one iteration from the next. They may omit colors entirely if they wish.
- ❄ A dendrite is a line segment that ends "in the air" without touching any other part of the crystal.
- ❄ Iterations may repeat. It is possible to spin a 5, three times in a row, for example.

Coach them to look for interesting phenomena:

- ❄ Sensitive dependence on initial conditions
- ❄ Simple building blocks leading to complexity
- ❄ Self-similarity
- ❄ Uniqueness within commonality
- ❄ Interesting mathematical patterns

Once every student has completed their snowflake, have them go around the room comparing their crystal with all the others, using tally marks to show same or different.

- ❄ Did we have any identical snowflakes?
- ❄ Could there ever be duplicate snowflakes?
- ❄ Is it likely?
- ❄ What would be the probability of two snowflakes being identical if we used just a single spin? (1/10) Two spins? (1/100) Three? (1/1000)

Finally, have them look at someone else's snowflake and attempt to work back through the sequence to determine which path was followed through the cloud.

Concepts in Chaos Theory

- ❄ Sensitive dependence on initial conditions – In order to predict the end result of certain processes, it is necessary to know the exact starting conditions. Often, it is not possible to measure those conditions with the precision required to make the desired predictions. Small differences in starting conditions can lead to radically different end-states.

- ❄ Simple building blocks leading to complexity – Very complex structures, like the genome of a human being, can be the result of using very simple building blocks, like the four DNA base pairs. There are many examples in nature of simple rules giving rise to complex systems.

- ❄ Self-similarity – Many shapes found in nature are "fractal". One characteristic of fractals is that they show a similar (or identical) pattern when viewed at very different scales.

- ❄ Uniqueness within commonality – Despite their uniqueness, at a certain level every snowflake looks like a snowflake. That is, there are underlying patterns to them. They arise from common building blocks and a common set of natural laws.