

Everything you wanted to know about coffee...

Newton's Law of Cooling states that the rate of change of the temperature of the coffee is proportional to the difference between the temperature of the coffee and the room (surrounding) temperature.

- (1) If  $y$  is the temperature of the coffee and  $T$  is the room temperature, express this as a differential equation.
  
- (2) Solve the DE in terms of constants  $k$  and  $T$ .
  
  
  
  
  
  
  
  
  
  
- (3) Assume that the room temperature is  $70^\circ\text{F}$  and the coffee is initially  $190^\circ\text{F}$ . After 3 minutes, the coffee is  $150^\circ\text{F}$ . Find the specific equation for the temperature of the coffee at any time  $t$ , where  $t$  is in minutes.
  
  
  
  
  
  
  
  
  
  
- (4) Write the differential equation using the specific values found for  $k$  and  $T$ .

Draw the slope field for the differential equation on your calculator. Draw a rough version below. (What's a good window?) Then graph your solution function (from 3) on your calculator to make sure it seems to be correct. (Does it match the slope field?) Draw it below as well.

- (5) Use Euler's method "by hand" with two steps to estimate the temperature at  $t = 1$  minute. (Make a chart or show the equations.)
- (6) Use your calculator program to check your work in (5). Is your work OK?
- (7) Find  $y(1)$  using your solution function. Which is higher – the estimate from Euler's method (5, above) or the real value? Explain why.
- (8) What if the value of  $k$  had been  $-.05$ ? Would the coffee cool more quickly or more slowly? What might have accounted for this different value?
- (9) Assume now that I just took a can of soda from the refrigerator. Would the differential equation look the same? Does anything change? Run the Euler program a couple of times with different initial conditions for the temperature of the soda. Sketch a graph of a typical solution curve.