Continuity and One-Sided Limits

A *continuous* function has a graph that can be drawn without lifting your pencil from the paper. There are various ways a function can fail to be continuous everywhere:

(A) Sketch the graph of $y = \dfrac{1}{x}$.

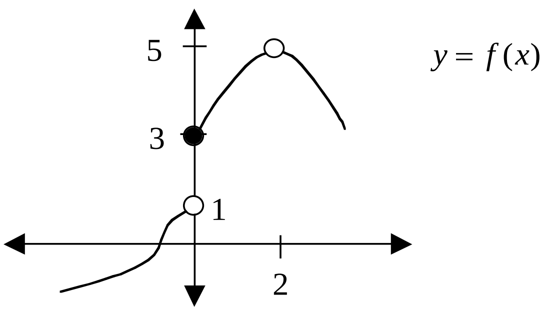
What happens to y as $x \to 0$ from the left?

What happens to y as $x \to 0$ from the right?

We write this as follows:

$\lim\limits_{x \to 0^-} \dfrac{1}{x} = -\infty$ and $\lim\limits_{x \to 0^+} \dfrac{1}{x} = \infty$

(B) Consider the following function $y = f(x)$:



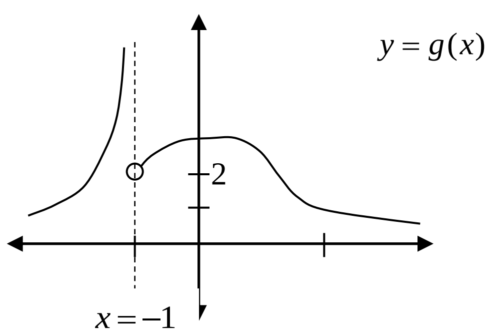
What happens to y as $x \to 0$ from the left?

What happens to y as $x \to 0$ from the right?

Write in limit notation:

$\lim\limits_{x \to 0^-} f(x) =$ _______ and $\lim\limits_{x \to 0^+} f(x) =$ _______

(C) Next, consider $y = g(x)$



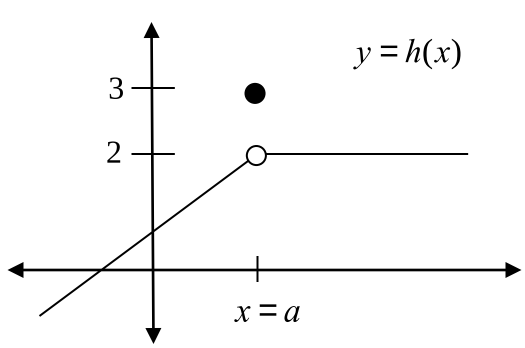
What happens to y as $x \to -1$ from the left?

What happens to y as $x \to -1$ from the right?

Write in limit notation:

$\lim\limits_{x \to -1^-} g(x) =$ _______ and $\lim\limits_{x \to -1^+} g(x) =$ _______

(D) Now, consider $y = h(x)$



What happens to y as $x \to a$ from the left?

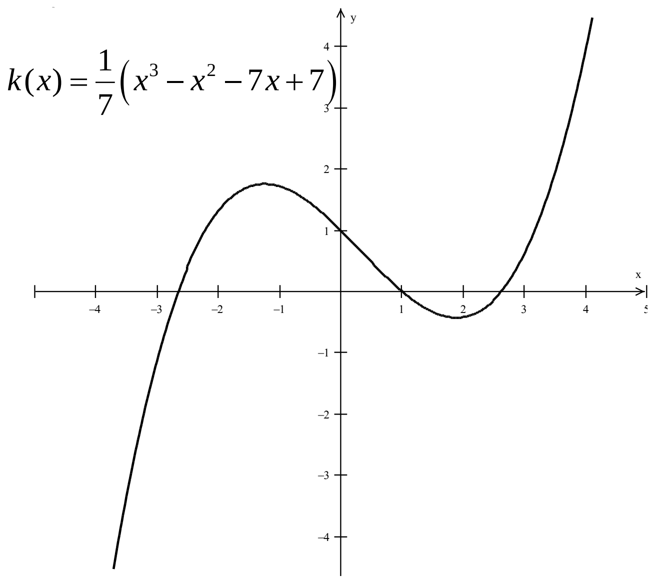
What happens to y as $x \to a$ from the right?

Write in limit notation:

$\lim\limits_{x \to a^-} h(x) =$ _______ and $\lim\limits_{x \to a^+} h(x) =$ _______

What are the conditions necessary for a function to be continuous at a point $x = a$?

Now, let's consider the graph of $y = k(x)$ below:



What happens to $k(x)$ as $x \to 0$ from the left?

What happens to $k(x)$ as $x \to 0$ from the right?

Write in limit notation:

$\lim_{x \to 0^-} k(x) =$ _______ and $\lim_{x \to 0^+} k(x) =$ _______

Evaluate $k(0) =$ __________.

Does $k(x)$ satisfy your conditions for continuity at the point $x = 0$?

For $k(x) = \frac{1}{7}\left(x^3 - x^2 - 7x + 7\right)$ above, evaluate each of the following:

$k(2) =$ $\qquad\qquad$ $k(3) =$

What does this tell you about one of the zeros of $k(x)$? Explain. Why is continuity important here?