2-28-2016

2016: "Analysis of the Effectiveness of Varying Car-sharing Business Models"

William Tong ’17
Illinois Mathematics and Science Academy

Sachin Govind ’16
Illinois Mathematics and Science Academy

Ankit Agarwal ’17
Illinois Mathematics and Science Academy

David Xu ’16
Illinois Mathematics and Science Academy, davidxu271@gmail.com

George Moe ’17
Illinois Mathematics and Science Academy

Follow this and additional works at: http://digitalcommons.imsa.edu/math_sw

Part of the Mathematics Commons

Recommended Citation
Tong, William ’17; Govind, Sachin ’16; Agarwal, Ankit ’17; Xu, David ’16; and Moe, George ’17, "2016: "Analysis of the Effectiveness of Varying Car-sharing Business Models"" (2016). Distinguished Student Work. 1.
http://digitalcommons.imsa.edu/math_sw/1

This Moody's Math Modeling Challenge is brought to you for free and open access by the Mathematics at DigitalCommons@IMSA. It has been accepted for inclusion in Distinguished Student Work by an authorized administrator of DigitalCommons@IMSA. For more information, please contact pgarrett@imsa.edu, jean@imsa.edu.
1 Summary

Our goal was to develop a model that can be used by car-sharing service companies to evaluate their business models and maximize their customer base. To do so, a model was created for each of the three major car-sharing service types in different city environments.

The first step to developing these models is to provide a more accurate representation of the driving habits of individuals. To do so, the population data is separated into bins determined by the factors of miles travelled and the amount of time that the user is in the car. Normally, the bins are separated into high, medium, and low based on the 30th and 70th percentiles (as can be seen in papers such as those by Fama and French), but another table with bins separated by 20 percent was created to gain a better distribution of the data. As expected, the greater majority of driving instances are either short distance trips that require little time or longer trips that require more time. From this a company is able to decide which service would be optimal to cater for the needs of their consumer base.

We then modeled each city topologically and figured out what cities had the most potential to implement a model. We determined that Richmond Virginia was the best place to implement a new solution because of its currently unoccupied area. We then conducted a Monte Carlo simulation that reflected the normal interactions between users and transportation within a city containing car-sharing businesses. From this we were able to determine which program(s) was the most popular within a normal city. We determined that the Round Trip business model would be the most popular within a generic city.

Finally, in order to account for increases in self-driving cars and green-tech cars, we modified the simulation to assume that all service cars were self-driving and made use of clean energy. As a result, the utilities of each car-sharing plan was altered, accommodating for the ability of the car to arrive at the user’s doorstep, thus removing the negative utility of having to walk to the station, and increasing the utility of using a green-tech car. We determined that the One-Way business model is the best model to implement in a generic city. Therefore, we conclude that the One-Way model is the best suited model to implement in Richmond, Virginia. In addition, the One-Way model is the ideal service for any business within a city that focuses on self-driving and/or green-tech cars. We recommend that a company start with a Round Trip model and monitor the rise of self-driving/green vehicles within the area. From there they may decide to switch to the One-Way model.
Analysis of the Effectiveness of Varying Car-sharing Business Models

Team 7789

28 February 2016
Contents

1 Summary of Work

2 Introduction and Outline
   2.1. Background Information
   2.2. Restatement of Problem
   2.3. Four Car-sharing models

3 City Structure and Layout
   3.1. Geometric Simplification of Cities
   3.2. Monte Carlo Simulation of Cities
   3.3. Inspection and Ranking of Cities

4 Who’s driving?
   4.1. Assumptions and Justifications
   4.2. Sample Data for 2009
   4.3. Mathematical Model through Simulation

5 Zippity Do or Don’t
   5.1. Assumptions and Justifications
   5.2. Mathematical Model
   5.3. Utility Functions
   5.4. Results

6 Road Map to the Future
   6.1. Modifications to the Model
   6.2. Results

7 Discussions
   7.1. Strengths and Weaknesses

8 Conclusions

9 References
2 Introduction

2.1 Background

A continually evolving field, the automotive industry consistently introduces a number of innovative technologies and services to ease the problem of transportation. One such service is termed Car-sharing. Car-sharing allows users to rent vehicles and use them for a short period of time without worrying about the additional costs associated with maintenance, fuel, and pollution, presenting a simple alternative to owning a car. Still an emerging concept, Car-sharing requires a great deal more analysis to fully understand the nuances and implications behind its implementation.

2.2 Restatement of the Problem

To more comprehensively understand the details of Car-sharing, we developed models to address the following three issues:

1. Considering that the two main factors influencing a driver’s decision to choose Car-sharing are the amount of time in which the car is used and the miles driven, how many Americans drive their vehicles for a given amount of time and miles? Breaking the two factors into low, medium, and high quantities, what percent of Americans fall into each combination of time and mileage?

2. What car share model applies best in Poughkeepsie, New York, Richmond, Virginia, Riverside, California, and Knoxville, Tennessee? How might the presence of Zipcar, a leading car share business, in each of these towns impact our analysis?

3. Given the growing trends in greener automobiles and self-driving systems, how might emerging automotive technology influence our analysis from the previous part? How might our rankings for each town be different?

2.3 Car-sharing Models

There are three major types of car-sharing models: round-trip, one-way, and fractional ownership Car-sharing.

1. Round-trip Car-sharing businesses run a car rental service with rental periods lasting by the mile, hour, or day, or even a combination of the three. During this rental period, the customer retains control of the car, but must return it to its starting point at the end of the session.
2. One-way car-sharing business models, much like bike-sharing programs, allow the customer to rent the car at one station and return it to any other existing partner stations. Depending on the service, the customer can either drop the car off directly at the station or to a designated location in which a jockey would relocate the car.

3. Fractional ownership follows a more literal definition of “car-sharing” where multiple people pool their resources and jointly purchase a single car. They then share the rights and usage of the car.

3 City Structure and Layout

3.1 Geometric Simplification of Cities

To approximate the behavior of the four given cities – Knoxville, Riverside, Richmond, and Poughkeepsie – we first approximated the shape and size of each city with geometric shapes. The results of these simplifications are shown below. From here onwards, all simulations and models are based upon these approximations.
3.2 Monte Carlo Simulation of Cities

Each city layout was divided into a grid using approximate land areas. Then, using publicly available data, we plotted the locations of existing Zipcar stations. We then generated 100 people at each grid point, each with their own desired location within the city. Then, we computed the distance to each desired location and each station. Using these, we were able to take the corresponding ratio and essentially compute the “probability” than a person at the given grid point went to a station instead of walked to their desired location. Then, these values were averaged for the 100 people at each grid point. We plotted the data using a multi-colored heat map on the map of the cities. The results are shown below:

These heat maps have been colored so that a red tone indicates a high probability of using a Zipcar, while a blue tone indicates a higher probability of walking. This provides very unique insight into the workings of each city: areas on the maps with darker, bluer regions would benefit from a new Car-sharing station for a competing company. Thus, we can recommend, with good confidence, the most effective places in each town to place a new Car-sharing station.
3.2 Inspection and Ranking of Cities

Using this data, it is clear that the areas with the largest blue areas would benefit the most from Car-sharing stations. Thus, the ranking system for cities we will use will be the cities with the largest blue areas. By visual inspection, the ordering of cities that would benefit most from new Car-sharing stations would be as follows:

1. Richmond
2. Knoxville
3. Riverside
4. Poughkeepsie

In the following sections, we will investigate this further and will determine which model is most appropriate for each city through more simulations. First, we will examine the demographics of drivers.

4 Who Is Driving?

Data from the National Household Travel Survey was used to build a table recording percentages of US drivers categorized by the amount of time spent driving and miles driven per day.

4.1 Assumptions and Justifications

Assumption: Pre 2009 Data can create a model relevant to present day

Justification: In general, since there has not been a significant event since 2009 that would have changed automobile time and distance, we are able to create a time progression model that can estimate the value of the percentages at 2016.

Assumption: The frequency of trips of a certain time are positively correlated to the frequency of drivers conducting a trip of that length.

Justification: For each trip there will be a driver, so as the frequency of trips rise, so does the number of drivers conducting such a trip.

Assumption: What is considered low, medium, and high is based off of a 30-40-30 split of the histogram for all trips.
Justification: This type of split is a standard split that approximates roughly one-third of the data for each type.

Assumption: Outliers can be removed.
Justification: Due to the nature of the dataset being used, empty responses were replaced with 999,999. Thus, the removal of these values would not influence the integrity of the analysis.

Assumption: Median is a good representation of the set.
Justification: Because the data is unimodal, the median is a serviceable representation of the values.

Assumption: Data from 2009 is a good approximation of the conditions in present day
Justification: After running the analysis on data from the year 2001 and 1985, there was no significant difference between the results from 1985, 2001, and 2009. Therefore, there will likely be little difference between 2009 and 2016.

4.2 Sample Data for 2009
The tables below shows these results. Because the medians and the standard deviations across all three years are roughly the same, there would likely be little difference between the data from 2009 and present conditions in 2016.

<table>
<thead>
<tr>
<th>Year</th>
<th>Median Trip Distance</th>
<th>Median Travel Minutes</th>
<th>Standard Deviation Trip Miles</th>
<th>Standard Deviation Travel Minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>3</td>
<td>10</td>
<td>4.684</td>
<td>12.253</td>
</tr>
<tr>
<td>2001</td>
<td>4</td>
<td>15</td>
<td>4.758</td>
<td>13.640</td>
</tr>
<tr>
<td>2009</td>
<td>4</td>
<td>15</td>
<td>4.854</td>
<td>13.720</td>
</tr>
</tbody>
</table>
4.3 Mathematical Model

To extract the table modelling the percentage of Americans in each category of time and mileage, raw data from the National Household Travel Survey (NHTS) was analyzed. The percentage of trips that fell within a certain range of time and distance were shuffled into their appropriate slots in the table.

In addition to the three by three table with the 30-40-30 percentile split, we also constructed a five by five table arranging values within each 20-percentile chunk in order to create a representation of the data with a finer resolution. Such a model is justified using a direct injection of data into a table is more accurate than an intermediary, approximating function to generate these values. The tables below show the frequency of each group of drivers.

<table>
<thead>
<tr>
<th>Time (min) Percentiles</th>
<th>Distance Traveled Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-30</td>
</tr>
<tr>
<td>0-30</td>
<td>0.2362</td>
</tr>
<tr>
<td>30-70</td>
<td>0.1255</td>
</tr>
<tr>
<td>70-100</td>
<td>0.0230</td>
</tr>
<tr>
<td>Totals</td>
<td>0.3918</td>
</tr>
</tbody>
</table>

Note that the higher percentage of drivers are concentrated along the main diagonal from the upper left to bottom right corners, which makes sense considering that a longer travel time suggests a farther destination.
5 Zippity Do or Don’t

To determine the optimal car share model for a given city, a simulation was built modelling the behavior of people traveling around a city. By measuring the transportation choices of the individuals based on utility functions, conclusions regarding the popularity of that car share option were drawn.

5.1 Assumptions and Justifications

Assumption: The user knows the nearest stations and how many cars are at each station.
Justification: Given the prevalence of smartphones equipped with car-rental service apps, it is not unlikely that car share users would be able to find information on the nearest stations.

Assumption: Driving is 10 times faster than walking.
Justification: The average walking speed is measured to be 3.1 miles per hour whereas the average driving speed in a city is 29.92 miles per hour, yielding a ratio of approximately 9.65 (insert citations). For the purpose of convenience, this value was rounded up to 10.

Assumption: The time required to park is not included in a person’s conception of utility.
Justification: In urban areas, parking spaces are ubiquitous and thus, are not typically factored into considerations regarding transit time.

Assumption: Traffic density plays no role in determining transit speed.
Justification: The speed used in determining utility and in building the simulation was the average driving speed in a city throughout the entire day. Therefore, it captures a reasonable picture of transit time both in times of high and low traffic densities.

Assumption: A city is a perfect grid without possibility for car crashes or other traffic accidents.
Justification: Cities are typically laid out in a grid-like fashion. Accidents are usually rare enough as to be negligible in the context of the simulation.

Assumption: The users act rationally and will always operate according to the greatest utility.
Justification: People tend to act rationally.
Assumption: Each map for each city can be represented by the given maps above (see City Structure and Layout).
Justification: Each map for each city represents the actual city in some sort of fashion and major geographical features are kept similar.

Assumption: Each city has a uniform population density
Justification: Since each of these cities is relatively urban, the population density throughout the city will not be skewed since the entire city will tend to be crowded with attractions.

Assumption: The difference between the one-way Car-sharing floating mode and the one-way Car-sharing mode is negligible.
Justification: Both models of Car-sharing are identical except that a jockey may reposition the car at the station for the former. Thus, from the point of view of the consumer, both systems are identical. Therefore, whether a company employs either model, the participation from the user is unaffected.

Assumption: Charge is not a factor in determining utility.
Justification: Given that all car share models would cost approximately the same amount, charge would not significantly impact a consumer’s choice.
5.2 Mathematical Model

We developed a Python program to simulate our situation. The program takes into account various factors of the city and runs a Monte-Carlo simulation. This type of simulation extracts workable results from a system based on random conditions with high data levels. Our program simulates user interactions in a city by creating a virtual city grid with users choosing various car-share rides in the city. This city grid was generated from a complete two-dimensional grid, with the appropriate number of edges removed to simulate city street topology. The users are randomly generated around the city with random destinations, and each judges the utility of each choice based on derived equations and optimal routing algorithms based on Dijkstra’s minimum path algorithm. After a user decides on a plan, it continuously updates its plan and may compete with other users. The users respond to this by considering their choices at each time step; their overall behavior and preferences illustrate which options are overall most viable in a city. By modeling the competitive environment between users and a limited supply of cars at each location in real time, our model presents a unique and realistic analysis of car sharing in cities. These results are shown in the table in section 5.4.

We approximated each city by categorizing them by population density. Poughkeepsie is a typical small city and Knoxville is a typical medium city. Richmond and Riverside did not differ significantly in their population densities, and were therefore modeled both as large cities.

5.3 Utility Functions

In our functions, the utility that the individual derives from a particular travel plan is negatively related to the amount of time required to reach the destination. As the travel time increases, the utility decreases. Thus, if there are two travel methods, the one that requires less time to travel has a higher utility. Within the simulation, the speed of walking was set to $\frac{1}{10}$ grid length per timestep while the speed of driving is 1 grid length per time step. In the following notation, $i$ represents the person’s initial point, $f$ represents the desired end location, and $s$ represents the location of the station. Subscripts on these stations imply closest and next closest stations.

1. The utility function for simply walking to the destination is

   \[ \text{Utility} = -10 \text{Distance}_{i \rightarrow f} \]

   which is simply the quotient between the walking distance and the speed of walking.

2. The utility function for a customer using a round-trip car sharing business option is

   \[ \text{Utility} = -10 \text{Distance}_{i \rightarrow s} - \text{Distance}_{s \rightarrow f}. \]
The first section is the time required to walk to the closest car-sharing service station and the second section is the time required to drive from the station to the intended destination.

3. The utility function for a customer using a one-way car-sharing business program is

\[ \text{Utility} = -(10 \text{Distance}_{i \rightarrow s_1} + \text{Distance}_{s_1 \rightarrow s_2} + 10 \text{Distance}_{s_2 \rightarrow f}). \]

The first section represents the time required to walk to the first station, the second section represents the time required for the customer to drive from one station to the next, and the final section represents the time required to walk from the drop-off station to the intended destination.

4. The utility function for a person jointly owning a car is

\[ \text{Utility} = -\left(\frac{1}{\tau} \text{Distance}_{i \rightarrow f} + 10 \left(1 - \frac{1}{\tau}\right) \text{Distance}_{i \rightarrow f}\right) \]

where \(\tau\) represents the number of co-owners of the car. The fraction \(\frac{1}{\tau}\) signifies the stake the driver has in the car, and thus the probability that the car is available to the user. This probability is then multiplied by the expected required walking time. The complement of this probability is then multiplied by the expected required driving time. The final utility is the average of the two partial times.
5.4 Results

Using Python’s built-in libraries for Graph Theory, we visualized the continuous process that comprised our simulation and model. Below are screen captures of the visualization of the simulations.

In the final output pictures, blue dots represent share car company cars, black dots represent privately owned cars from the fractional ownership model, dark green dots represent stations, light green dots represent consumers, blue lines represent a connection between the consumer and the destination, pink lines connect consumers to their car drop-off point in the round-trip model, and orange lines connect consumers to their car drop-off point in the one-way trip model. Our results from the simulation are summarized in the table below:

<table>
<thead>
<tr>
<th>City Size</th>
<th>Walk</th>
<th>Round</th>
<th>One-Way</th>
<th>Fractional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>0.28%</td>
<td>58.83%</td>
<td>37.97%</td>
<td>2.92%</td>
</tr>
<tr>
<td>Medium</td>
<td>0.43%</td>
<td>67.98%</td>
<td>27.99%</td>
<td>3.60%</td>
</tr>
<tr>
<td>Small</td>
<td>1.38%</td>
<td>84.08%</td>
<td>8.51%</td>
<td>6.03%</td>
</tr>
<tr>
<td>Total %</td>
<td>0.70%</td>
<td>70.30%</td>
<td>24.82%</td>
<td>4.18%</td>
</tr>
</tbody>
</table>
Note that the round-trip held the overwhelming majority but with one-way still a substantial fraction of the total decisions. Thus, it can be concluded that the round-trip model would be the preferable car-share option to choose for all cities.

6 Roadmap to the Future

6.1 Modifications to the Model

With growing trends in autonomy and green energy, the attractiveness of Car-sharing may increase significantly. Given that about 82 percent of Americans conscientiously make greener choices when buying products, the utility functions as autonomous, cleaner choices appear in the car share industry will likely change significantly. In addition Car-sharing with autonomous cars will allow for cars to essentially drive up to their customer and pick them up. Therefore, it is assumed that all the cars within the model would be self-driving and green and we adjusted our utility function accordingly:

1. The new utility function for simply walking to the destination is

   Utility = $-10 \text{Distance}_{i\rightarrow f}$

   This clearly does not change between the two models

2. The new utility function for One-Way car-shares is:

   Utility = $-0.82(\text{Distance}_{i\rightarrow s} + \text{Distance}_{s\rightarrow f})$

   Since the car will come to you and then you would drive the car the utility function only correlates with the distance from the car to you and the distance from you to the car. In addition we multiply by a factor of 82 since 82% of Americans would make greener choices.

   This would lower the utility further since it would be considered a better choice.

3. The new utility function for round-trip car-shares is:

   Utility = $-0.82(\text{Distance}_{i\rightarrow s} + \text{Distance}_{s\rightarrow f})$

   Once again, the car will come to you so the walking distance to the nearest station now become driving distance from the station. Finally we multiply by a factor of 0.82 once again for the same reason as before.

4. Finally, the new utility function for Fractional Ownership is:

   Utility = $-\left(\frac{0.82}{\tau}\text{Distance}_{i\rightarrow f} + 10\left(1 - \frac{1}{\tau}\right)\text{Distance}_{i\rightarrow f}\right)$
Since having a car co-owned is the same as self-driving a car if the car is on, then there would be no effect on the distance traveled via the car. The only thing that would change would be the driving aspect. Therefore we multiplied only the driving part of the probability by 0.86.

### 6.2 Results

The Monte Carlo City Simulator returned significantly different results after accounting for self-driving and eco-friendly cars. Our simulation generated 35 simulations of car-sharing interactions spread across each city. The results for problem three are summarized in the table below:

<table>
<thead>
<tr>
<th>City Size</th>
<th>Walk</th>
<th>Round</th>
<th>One-Way</th>
<th>Fractional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large</td>
<td>14.97%</td>
<td>0.00%</td>
<td>62.32%</td>
<td>20.46%</td>
</tr>
<tr>
<td>Medium</td>
<td>7.90%</td>
<td>0.02%</td>
<td>89.01%</td>
<td>2.60%</td>
</tr>
<tr>
<td>Small</td>
<td>4.24%</td>
<td>0.00%</td>
<td>95.09%</td>
<td>0.50%</td>
</tr>
<tr>
<td>Total</td>
<td>0.09%</td>
<td>0.00%</td>
<td>0.82%</td>
<td>0.08%</td>
</tr>
</tbody>
</table>

In contrast to the results for problem 2, note that the one-way car-share model now holds the overwhelming majority. From a practical consideration, in a situation where cars may be able to drive independently, the ability for a car to be able to follow the driver would be make the one-way car share models the most ideal—no longer must the driver bother walking to stations or returning the cars back to their original posts.
7 Discussion

In the following section, we discuss the various strengths and weaknesses of our models. When modeling scenarios, it is crucial to evaluate how effective each solution is in the context of the original problem.

7.1 Strengths and Weakness

Strengths:

1. Because the data was used directly to calculate the percentages without use of an intermediary approximating function, our model from the first question is fairly accurate.
2. Our Python simulation is relatively flexible and allows for a wide variety of city types, configurations, and attributes.
3. The constants used in the utility functions were based off of real data, they reflect the real world with reasonable accuracy.
4. The maps for the four cities were based off the real cities and included the actual Zipcar station locations. This allowed us to estimate the rankings with greater accuracy.

Weaknesses:

1. Because population density data from within a city was inaccessible, the model from the second question assumes that population density is approximately even throughout a city.
2. Due to computational and practical concerns, the Python simulation is unable to completely simulate their counterparts in the real world.
3. Given the significant addition of computational expense involved with modelling traffic patterns in each city, the Python simulation does not incorporate specific nuances in travel speed due to traffic conditions and, instead, takes the average speed in a city.
8 Conclusions

In conclusion, we have modeled with reasonable accuracy the evolving car-sharing industry. First we were able to properly model the percentage of US drivers who drive low, medium, and high distances for low, medium, and high periods of time, helping car-sharing companies decide which drivers to target and how to distribute their vehicles to cater to one or more of these driver types. Second, we modelled the usage of various car-share options throughout different cities in order to determine which model functioned best in which city. In order to determine each city ranking, we modeled the area of each city with Zipcar locations defined throughout the city. We found that Richmond contained the most area in which a new model could be implemented and consequently, it was the most feasible option to compete against Zipcar. We ultimately determined that round trip is the best model to use in any city by judging it against a generic square city. Therefore we conclude that the round trip option should be implemented in Richmond, Virginia. Finally, we modeled the case in which self-driving and green cars become increasingly involved in society. Through our model, we saw that the one-way option (either floating or station) is the most feasible to guarantee a large amount of participation from the community. We recommend assessing company costs to determine whether to use the floating or the station option. Since the ranking system does not change in either of these situation, we concluded that the overall best business choice to make is to implement the one-way option in Richmond, Virginia.
9 References


