BC 3 Name:________________________________

Geometric Series

Defn: An infinite series is an expression of the form

$$a_1 + a_2 + a_3 + \ldots + a_k + \ldots \quad \text{or} \quad \sum_{k=1}^{\infty} a_k \quad \text{or simply} \quad \sum a_k$$

The problem is to add up an infinite number of terms to find the value of the sum. Sometimes, it's fairly easy:

$$.333333.... = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \ldots = \frac{1}{3}$$

This type of series may look familiar. It's called a geometric series.
Let's first look at the <u>n^{th} term</u>. Complete the following:

<u>Recursive form</u>	<u>Closed (explicit) form</u>
$a_1 = a_1$	$a_1 = a_1$
$a_2 = a_1 r$	$a_2 = a_1 r$
$a_3 = a_2 r$	$a_3 = a_1 r^2$
$a_4 =$ ____	$a_4 =$ ____
…	…
$a_n =$ ____	$a_n =$ ____

Sum of a Finite Geometric Series

Let $S_n = a_1 + a_1 r + a_1 r^2 + \cdots + a_1 r^{n-1}$

Multiply each term above by r, and vertically line up terms similar to the terms above, according to the power of r.

$r \cdot S_n =$

Subtract the second line from the first, vertically, according to similar terms.

$S_n - r \cdot S_n =$

Factor the left side and solve for S_n.

(1) Find $\sum_{j=1}^{8} 3^j$

(2) Find $\sum_{j=1}^{6} 2 \cdot \left(\frac{1}{3}\right)^{j-1}$

Infinite Geometric Series

Given $S_n = \dfrac{a_1(1 - r^n)}{1 - r}$, what happens to the <u>formula</u> for S_n when $r = 1$?

What does the actual series become when $r = 1$? Write out a few terms.

As $n \to +\infty$ with $|r| > 1$, what happens to $|r^n|$? Does the series converge or diverge?

As $n \to +\infty$ with $|r| < 1$, what happens to r^n?

In terms of a and r, what is $\lim\limits_{n\to\infty} S_n$?

Thus, for an infinite geometric series with $|r| < 1$ and first term $a_1 = a$, we find

$$S = \sum_{n=1}^{\infty} a\,r^{n-1} = \frac{a}{1-r}.$$

Find:

(3) $6 + 4 + \dfrac{8}{3} + \ldots$

(4) $12 - 9 + \dfrac{27}{4} - \ldots$

(5) $\displaystyle\sum_{j=1}^{\infty} \frac{3}{5^j}$

(6) $\displaystyle\sum_{j=0}^{\infty} 4 \cdot \left(\frac{2}{3}\right)^j$