Derivatives of Exponential Functions

This activity sheet is designed to find the derivative of the function \( f(x) = b^x \).

(1) Graph each function (---) and its derivative (- - -). Try the window \([-2, 4] \times [-2, 20]\).

\[
\begin{align*}
  f(x) &= 2^x \\
  g(x) &= 3^x \\
  h(x) &= e^x
\end{align*}
\]

(2) What can be said about \( h \) and \( h' \)?

(3) To investigate the derivative of \( f(x) = b^x \), we consider the ratio of the derivative to the function itself. In other words, we want to consider the set of ordered pairs of the form \((b, \text{ ratio of } y'/y)\) for a variety of values of \( b \).

Begin with the specific case where \( b = 2 \). On the "y=" screen, let :

- \( y_1 = 2^x \), \( y_2 = nDeriv(y_1(x), x) \), and let \( y_3 = y_2(x)/y_1(x) \) OR
- \( Y_1 = 2^X \), \( Y_2 = nDeriv(Y_1, X, X) \) and \( Y_3 = Y_2/Y_1 \)

Use "Table." What is true about \( y_3 \)? What is its value?

Change \( y_1 \) and repeat this process for each of the values of \( b \) given below. Fill in the ordered pairs with \( y_3 = \text{ratio of } y'/y \). Plot these ordered pairs on the graph.

\[
\begin{align*}
  (2, \_\_\_), (0.25, \_\_\_\_\_\_\_\_) \\
  (0.5, \_\_\_), (1.5, \_\_\_\_\_\_\_) \\
  (3, \_\_\_), (4, \_\_\_\_\_\_\_) \\
  (5, \_\_\_), (6, \_\_\_\_\_\_\_) \\
  (7, \_\_\_), (8, \_\_\_\_\_\_\_\_\_\_\_) \\
\end{align*}
\]
What does this function look like?

In short, we have the relationship \((b, \underline{\text{___________}})\). This means that the ratio of \(\frac{d}{dx}(b^x) = \underline{\text{___________}}\), so the derivative of \(f(x) = b^x\) is \(\frac{d}{dx}(b^x) = \underline{\text{___________}}\).

(4) We’ve seen that the derivative of \(y = b^x\) is a multiple of the function itself. Show this by setting up the derivative by definition and factoring out \(b^x\). The remaining expression may be seen as the derivative of \(\underline{\text{_______}}\) at \(a = \underline{\text{_______}}\).

(5) What does your derivative rule imply about the derivative of \(y = e^x\)? Show.

(6) Find the derivative of each of the following using the rule found above. (For some, you will want to rewrite the expression using properties of exponents.)

\[ y = 10^x \quad y = 4 \cdot 3^x \quad y = 4^{2x} \]

\[ y = (1/2)^x \quad y = \frac{3}{5^x} \]

\[ y = x^2 + 2^x + 2 \quad y = 3 \cdot 6^{-x} \]

\[ y = e^{2x} + x^2e \quad y = 3e^{x+2} + 5^{x-1} \]