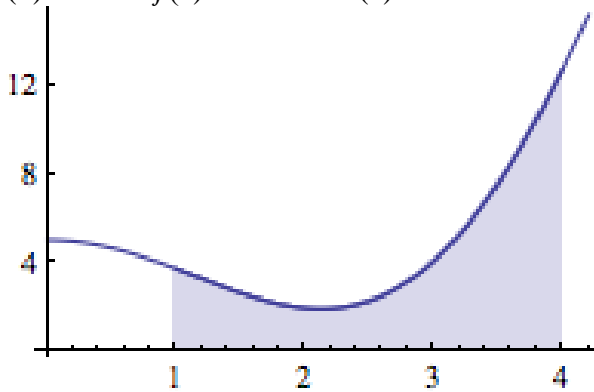


(1) Let  $f(x) = x^2 + 5\cos(x)$  on the interval  $[1, 4]$ .



Use the Riemann program to complete the table below. Give 3 decimal places.

number of rectangles	Left-Hand Endpoint	Midpoint	Right-Hand Endpoint
$N = 5$			
$N = 10$			
$N = 20$			
$N = 40$			

Does the convergence improve in all three cases above as  $N$  grows larger? Are there any exceptions to this?

(2) Let  $g$  be a strictly increasing function. Draw pictures with left- and right-hand rectangle approximations. What can be said about the relationship between the values of the LH, MP, and RH approximations over a given interval  $[a, b]$  for any given number of rectangles? (Give an inequality.)

Continuing with our increasing function  $g$ , we want to consider concavity. Draw pictures with  $g$  concave up and with  $g$  concave down. Does concavity affect whether the approximation is too high or too low? Explain.

- (3) Now assume that  $h$  is a decreasing function over an interval  $[a, b]$ . What is the relationship between the values of the LH, MP, and RH approximations?

Will concavity have an effect on this relationship?

- (4) If  $j$  is a linear function, what can be said about the relationship between the values of the LH, MP, and RH approximations for any given number of rectangles? Draw a picture to illustrate this.

To demonstrate this further, choose a linear function  $j$ , an interval, and a value of  $N$ . Find the three approximations (LH, MP, RH) with your calculator program. Confirm that the relationship you stated above does actually hold in this example.