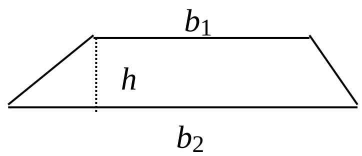
BC 2 Name:____________________________

Approximations 4

Trapezoidal Rule

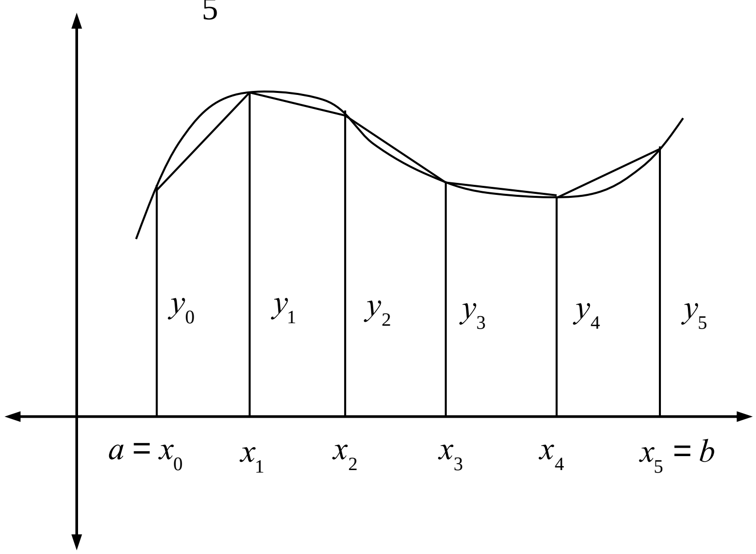
On the last couple of sheets, we used rectangles to approximate the area under the curve. We now consider trapezoids as another method of approximation.

First, recall the formula for the area of a trapezoid.



A(trap) =

As before, we partition the interval [a, b], but now, we form trapezoids by using the heights of the function at each of the endpoints of the subdivisions. Here, we use n = 5 subintervals with $\Delta x = \dfrac{b - a}{5}$.



Complete expressions for the following areas as started.

Area (under f)

$\approx$ A(1st trap) + A(2nd trap) + A(3rd trap) + A(4th trap) + A(5th trap)

$= \dfrac{1}{2}\left(y_0 + y_1\right) \cdot \Delta x$ +

Factor out the common terms 1/2 and Δx and then simplify:

=

Generalize this for n trapezoids. (Use "...", y_{n-1}, and y_n.)

(1) Let $f(x) = x^2 + 1$ on $[0, 2]$ with $N = 4$.
Write out the sum for the trapezoidal approximation and then evaluate the sum.

Now, with the same f and N, find the LH and RH approximations with the Riemann program.

LH = RH =

How do the values for LH and RH compare to the trapezoidal approximation?

Illustrate and explain why is this true geometrically.

Now show this relationship algebraically. (Write out LH and RH and add.)

(2) If g is a linear function, what is significant about the trapezoidal approximation?

(3) Let f be a function that is concave up on the interval $[a, b]$. What, if anything, may be said about the relationship between the trapezoid approximation and the actual area over $[a, b]$ for any number of subintervals?

What if f is concave down on the interval $[a, b]$?