BC 1 Name: ____________________________________

Limits 5 **Limits and Continuity (formal)**

The concept of limit allows us to replace our informal definition of continuity with one that is mathematically precise.

Definition: A function f is continuous at a point $x = a$ if

(1) $f(a)$ exists,

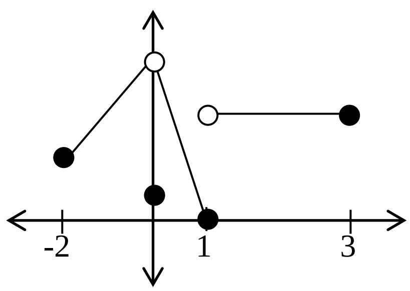
(2) $\lim_{x\to a} f(x)$ exists, and

(3) $\lim_{x\to a} f(x) = f(a)$.

In other words, the function exists at the point, the limit exists at the point, and the functional value equals the limit at the point. If any of these conditions does not hold, we say the function is discontinuous at $x = a$.

(1) Determine any point(s) of discontinuity (x-values) for each of the following functions.

(a)



(b) $f(x) = \dfrac{x}{(x-1)(x+2)}$

(c) $g(x) = \dfrac{x}{x^2 + 3x}$

(2) Suppose $f(x) = \begin{cases} \dfrac{x^2-9}{x-3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$.

(a) Show that f is discontinuous at $x = 3$. <u>Justify</u> by checking each of the three conditions in the definition for continuity listed above.

(b) How should $f(3)$ be defined instead to make f continuous at that point?

(3) (a) Give an example of a function with a removable discontinuity (i.e., a hole) at $x = 2$.

(b) Give an example of a function with a jump discontinuity (i.e., a jump or a break forming two separate sections) at $x = 2$.

(4) Let $h(x)=\begin{cases}3ax+7, & x<-1\\ x^2+a, & x\geq -1\end{cases}$. Find the value(s) of a so that h will be continuous at $x = -1$. (Use the definition and limits!)

(5) Define the value of $g(-2)$ so that g will be continuous if $g(x)=\dfrac{x^2-3x-10}{x^2+2x}$.

(6) If $k(x)=\begin{cases}3x-2, & x<2\\ 5, & x=2\\ x^2, & x>2\end{cases}$, use the definition of continuity and limits to determine whether or not k is continuous at $x = 2$.

(7) Find each limit. Approximate and guess if necessary.

(a) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$ (b) $\lim\limits_{x\to\infty}\dfrac{e^x}{x}$ (c) $\lim\limits_{x\to\infty}\dfrac{\ln x}{x}$