BC 3 Name:_____
Series 3

It's time to generalize the process of finding an approximating polynomial. Assume we have an arbitrary function f, and assume that at $a = 0$, $f(0)$ and the derivatives $f'(0)$, $f''(0)$, ... , and $f^{(n)}(0)$ all exist.

Let $P_n(x) = C_0 + C_1x + C_2x^2 + C_3x^3 + ... + C_nx^n$ be an n^{th} degree polynomial.

If we set $P_n^{(k)}(0) = f^{(k)}(0)$ for $k = 0, 1, 2, ..., n$, we can solve for C_k, still for $k = 0, 1, ..., n$.

(1) Set $f(0) = P_n(0)$ to find C_0 in terms of $f(0)$.

(2) Find $P_n'(x)$. Then set $f'(0) = P_n'(0)$ and find C_1 in terms of $f'(0)$.

(3) Find $P_n''(x)$. Then set $f''(0) = P_n''(0)$ and find C_2.

(4) Find $P_n'''(x)$. Set $f'''(0) = P_n'''(0)$ to find C_3.

(5) Find $P_n^{(n)}(x)$ and set $f^{(n)}(0) = p_n^{(n)}(0)$ to find C_n in terms of $f^{(n)}(0)$.

(6) Write the polynomial $P_n(x)$ in terms of the derivatives of f evaluated at 0.

(This is called either the n^{th} order Taylor polynomial about $x = 0$ **or** the n^{th} order Maclaurin polynomial for the function f.)

Maclaurin Polynomial of degree n (Taylor Polynomial about $x = 0$)

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n = \sum_{k=0}^{n} \frac{f^{(k)}(0)}{k!}x^k$$

Taylor Polynomial of degree n about $x = a$

$$f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x - a)^n = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x - a)^k$$

(1) Use this process to find the Maclaurin polynomial for $f(x) = e^x$ when $n = 4$.

(2) Use this process to find the Taylor polynomial for $f(x) = \ln(x)$ about $x = a = 1$ when $n = 3$. (How does this compare to your work on Series 1?)