

**Part 1: Estimating  $\ln(10)$**

Again, we have  $\ln(1 + x) =$  \_\_\_\_\_ for  $(-1, 1]$ .

(a) Unfortunately, we can't use this to find  $\ln(10)$  because of the interval of convergence. However, we can rewrite  $\ln(10)$  as  $-\ln(1/10)$ . Then we may use the series above for  $\ln(1/10) = \ln(1 + x)$  by using  $x =$  \_\_\_\_\_. Write out 3 terms of the series and find their sum to approximate  $-\ln(1/10)$ . Also check  $\ln(10)$  directly on your calculator for comparison.

(b) Here's another approach. Write the series for the following:  
interval of convergence

$$\ln(1 + x) =$$

$$\ln(1 - x) =$$

$$\ln\left(\frac{1 + x}{1 - x}\right) =$$

What is the mathematical relationship between the interval of convergence of this last series above and the intervals for the first two?

We still want to find  $\ln(10)$ , so let  $\left(\frac{1 + x}{1 - x}\right) = 10$ . Find  $x$ .

Use this value of  $x$  with 3 terms of the last series above to approximate  $\ln(10)$ .

(c) Which approximation is better? faster?

**Part 2 : Estimating  $\sqrt{17}$**

(a) Find the binomial expansion (the Maclaurin series) for  $f(x) = \sqrt{1+x}$ .

(b) What is the interval of convergence? (Make a good guess! Graphing would help.)

(c) So what do we do to approximate  $\sqrt{17}$ ?

We write:  $\sqrt{17} = \sqrt{16\left(1 + \frac{1}{16}\right)} = \sqrt{16} \times \sqrt{1 + \frac{1}{16}} = 4\sqrt{1 + \frac{1}{16}}$ .

Now use this with the series above to approximate  $\sqrt{17}$ .