IMSA

Name:_____

Name:_____

Cycloids

Begin with a straight line and a disk. Consider a specific point on the disk. Roll the disk along the straight line. A cycloid is the curve created by tracing the path of the point.

Take the paper and draw a straight line on it. Draw this lengthwise, about one-fourth of the distance from one edge of the paper.

(1) Begin with a point <u>on the edge</u> of the disk. Roll the disk along the line and trace out the path of the point. Hold the disk and pencil carefully. Draw more than one complete revolution. Draw a small version of this below.

(2) Now consider a point <u>exterior</u> to the disk. (It's probably easiest to take a pencil and extend this along one radius of the disk so that it sticks out an inch or so beyond the edge of the disk. You may want to tape it in place.) Roll the disk and your pencil and trace out the path of the tip of your pencil. Draw more than one complete revolution. Draw a small version of this below. This is called a *prolate cycloid*.

(3) Next, consider a point on the <u>interior</u> of the disk. (Use your pencil to make a hole in your disk.) Roll the disk along the line and trace the path of the point. Draw more than one complete revolution. Draw a small version of this below. This is called a *curtate cycloid*.

Name:_____

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Now that you've looked at the basic graphs of three types of cycloids, let's concentrate on the first one you drew – the one with the point on the edge of the circle. Your next goal is to find the parametric equations of the curve given by this cycloid.

For this cycloid, assume that the radius of the disk is a units, and the angle of revolution is t radians. We'll also assume that at the beginning, the disk is sitting on the x-axis at the origin. Let the point to be traced start at the origin when $t = 0$.

Sketch a good graph of a circle that has rolled just a little ways along the positive x-axis. Show the tracing point on the circle and label t. Find the coordinates of this point in terms of t. (To make this simpler, you may wish to start with $r = 1$ and later modify your equations for $r = a$ units.)

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Writing Project: Due Friday, 6 December 2013, at the beginning of class.

What to do:

(1) An opening paragraph, including a description of what a cycloid is. (Do not copy the sentences on this sheet! Rephrase this in your own words.)
(2) An explanation to someone at your level of mathematics to show how you arrived at your parametric equations. This should be done in detail and with clarity. That is, the writing and the explanation are important.
(3) Include graphics. One graph should aid your explanation of the equations. Another should show the curve – more than one complete arch to show the pattern. More graphs are optional, but these may be helpful.
(4) Estimate either (1) the area under one arch for a specific value of the radius *a* OR (2) the length of the curve of one arch. (You may do either of these in many different ways, except that you may NOT use calculus! It is to be an estimate.) Explain (and possibly show) how you obtained this estimate.
(5) Describe briefly how the graphs change when the point is outside of the circle and then inside. How do the graphs vary? Also, are there other types of cycloids besides the three already mentioned? You may wish to describe these briefly as well.
(6) A closing paragraph – to summarize the ideas in your paper.

A few rules:

(a) Students are expected to work in pairs. (If you need a partner, please check with me.)
(b) Type! Double-space with reasonable margins and a reasonable 12-pt font.
(c) Use Equation Editor to create equations.
(d) Graphics may be done by hand, by transferring them from your calculator, or with computer software. With any of these, add labels by hand or by computer.
(If you want to use *Mathematica*, the command for plotting parametric functions is:

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ParametricPlot[{x(t), y(t)},{t,tmin,tmax}]
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Fill in your functions for *x* and for *y* and the appropriate window for *t*. Resize the graphic approximately in *Mathematica* first, and then use copy/paste. Resize a bit more as necessary in *Word*.) Of course, any graphics or any other information taken from other sources, websites, etc., should be clearly and correctly referenced.
(e) Submit one paper per pair to turnitin.com by late afternoon, Friday.
(f) Turn in these pages, the rubric, and your project on Friday, December 6, at the beginning of class. One copy per pair of students, please.

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Grading Rubric for Cycloids

(1) 20 pts	Both Introductory and closing paragraphs: Clear and correct explanation of a cycloid. Summary.	
(2) 30 pts	Quality of mathematical content: Correct explanations to derive equations. Correct work and explanation to approximate area under arch or arc length. Correct explanations of two variations of cycloids. More info?	
(3) 10 pts	Quality of graphics: Two graphics (or more), all with clear and correct labels and scales and references as necessary.	
(4) 25 pts	Quality of written explanations: Explanations are easy to read, coherent, and with complete sentences. Explanations show an appropriate sense of the audience.	
(5) 15 pts	Quality of overall project: Very few errors in mechanics. Typed, double-spaced, reasonable margins, 12 pt font. Entire product is a cohesive, single paper. Organization and flow lead to effectiveness.	
100	Total	