

GEOMETRY IN ART, ARCHITECTURE, AND NATURE

"Teach your scholar to observe...you will soon raise his curiosity. Put the problems before him and let him solve them himself. Let him know nothing because you have told him, but because he has learned it for himself. Undoubtedly the notions of things thus acquired for oneself are clearer and much more convincing than those acquired from the teaching by others ..."

Jean-Jacques Rousseau: *Emile* (1762)

- We all hope that students are able to see math everywhere around them every day...let's be honest- geometry is the easiest way to see and demonstrate that.

Kids as young as 2-3 can recognize and name shapes including hexagon, rhombus, diamond, octagon, etc.- There's an app for it!! They can often recognize it in every day objects.

- Older students can often get 'bored' doing proofs every day. Older students like to do fun and potentially competitive activities.

The Challenge

- 1) Take specific photographs, find images online, or create a piece of artwork that shows 10 (or some arbitrary number) specific geometric relationships.
- 2) Choose 3 (or any number you choose) of the relationships and provide the mathematical analysis of the image. Measurements can be done by hand with a ruler or protractor
- 3) Presentation in powerpoint, poster, artwork, etc. format. Can be hung around the classroom

The Challenge

- This is designed as a longer project- a couple of weeks, but can be easily altered for things like
 - In place of a quiz at the end of the unit
 - Introduction to a concept
 - Short class day scavenger hunt- maybe done in the classroom or outside during break
 - Parent and student together assignment- find the geometry at home type of thing
- The ability to do a presentation, poster, or art project allows all students of all ability levels to participate successfully
- Including Art and Nature allows the students to think beyond the typical architecture

The Concept Hunt

**These Geometry principles were selected at random

- 1) Two parallel lines cut by a transversal
- 2) Triangle inscribed in a circle
- 3) Congruent Triangles
- 4) A square, rectangle, and rhombus (non-square) showing their differences and similarities
- 5) A circle with its tangent
- 6) A regular polygon with more than five sides
- 7) Similar triangles
- 8) Two non-congruent polygons

9) A cone and cylinder showing their differences in area

10) A rectangular prism and rectangular pyramid showing the similarities in their bases

Other options:

- Relationship between a sphere and a cylinder of similar diameters

Two Parallel Lines Cut by a Transversal

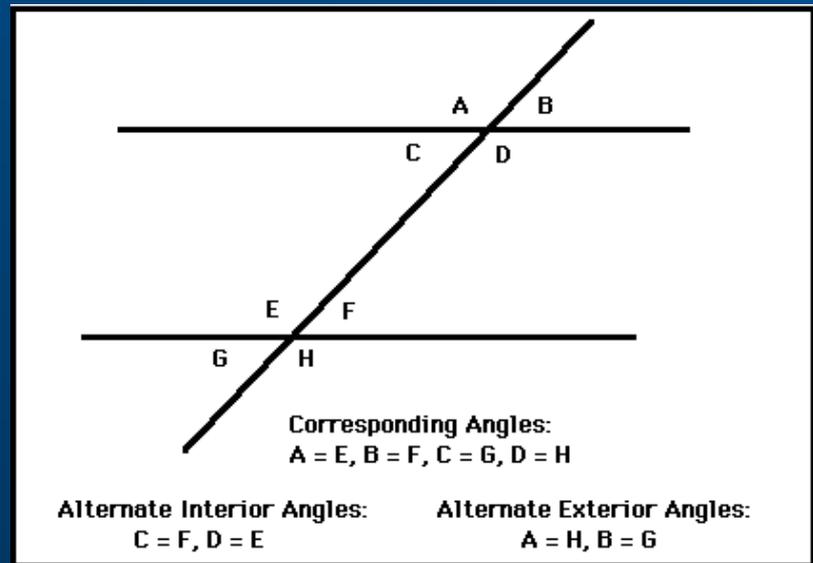
The Image: Hancock Building



Img src:
http://en.wikipedia.org/wiki/John_Hancock_Center

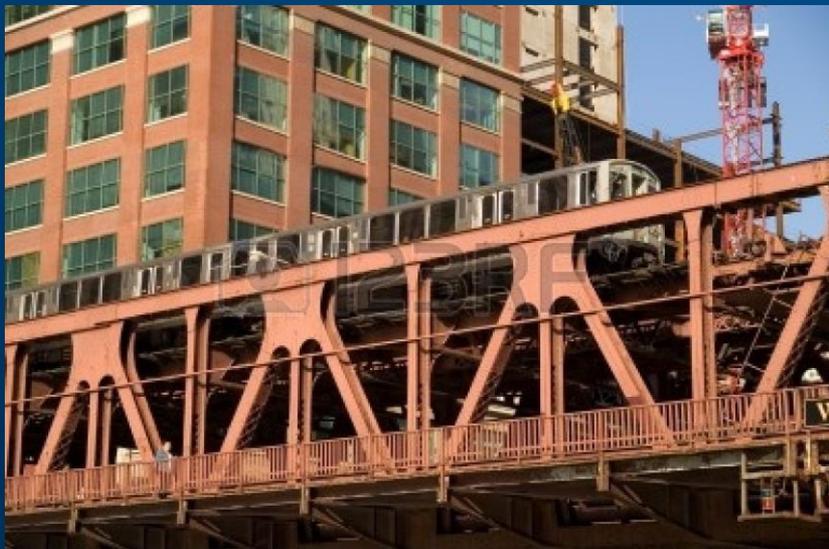
The Geometry Concept

Parallel Lines Conjecture
Alternate Interior Angles are
congruent



Congruent Triangles

Bridge over the Chicago River



Img Src: http://www.123rf.com/photo_2409205_double-decker-bridge-over-chicago-river-in-downtown-chicago.html

Congruency Postulates

- If the lengths of all three sides of two triangles are equal, then they are congruent.
- If two lengths are equal as well as the angle between them are equal in two triangles then they are congruent.
- If two angles and the side between them are congruent in two triangles then they are congruent.

Triangle Inscribed in a Circle

The Dollar Bill



Img Src:
<http://dcsymbols.com/notebook/geometer.htm>

Conjecture

- If all three points of a triangle touch the circle, then if you draw a line bisecting the sides of the triangle from the opposite angle and extending to the circle, from all three angles, these will all meet in the center of both the triangle and circle.

Regular Polygon with more than four sides

Pentagon, Washington, D.C.



Img Src: <http://pentagon.spacelist.org/>

The Geometry

- Finding the sum of interior angles of any polygon- $(n-2)180$ where n is the number of sides.
- If the polygon is equilateral, then divide the total sum by the number of angles.

Square, Rectangle, Rhombus

Frank Lloyd Wright's Robie House



Img Src:
http://en.wikipedia.org/wiki/Robie_House

The Concept

- Square- all sides are of equal length and all angles are 90 degrees.
- Rhombus- All sides are equal in length but angles are not all equal to 90 degrees.
- Rectangle- All angles equal 90 degrees but sides are not of equal length.

Similar Triangles

Hancock Building, Chicago



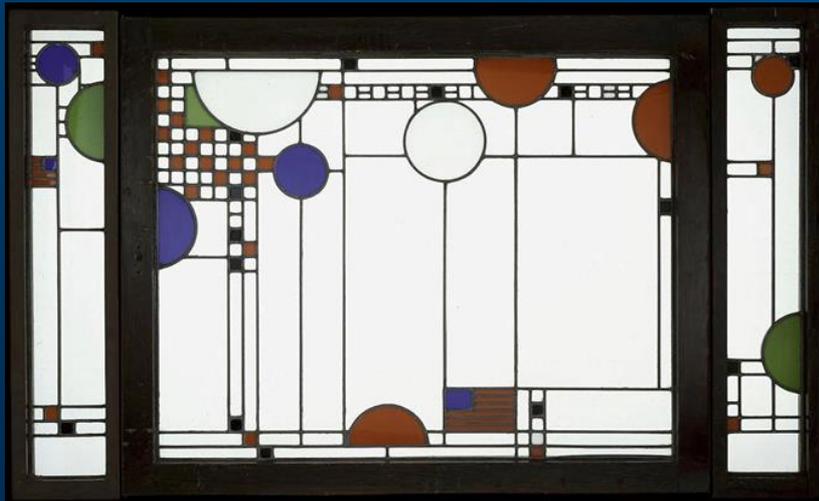
Img Src: <http://www.seechicagorealestate.com/chicago-streeterville-john-hancock-center-condominiums.php#.Uv6Fqs6K-4o>

Geometry Concept:

- Triangles are similar if we are able to find the scale factor or proportionality factor.
- In this image it is easy to see with many shared angles and sides.

Circle and Its Tangent

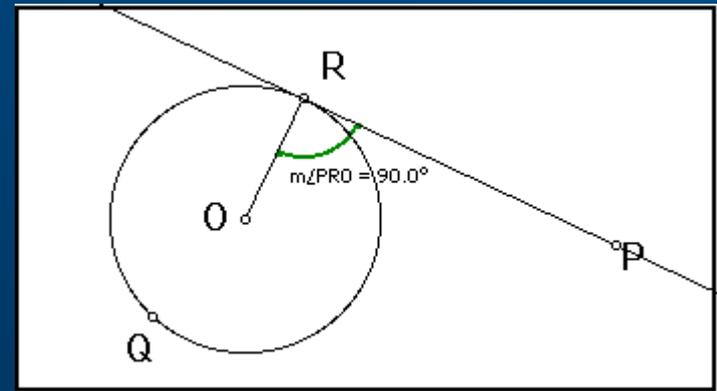
Frank Lloyd Wright- Avery
Coonley Playhouse-
Riverside, IL.



Img Src:
<http://www.artic.edu/aic/collections/artwork/105203>

Tangent Conjecture

Any tangent line to a circle is perpendicular to the radius drawn to the point of tangency.



Two Non-Congruent Polygons

Michigan Ave Bridge



Img Src: <http://www.cityprofile.com/illinois/photos/4940-chicago-michigan-avenue-bridge1.html>

Geometry Concept

- The challenge is to find two non-congruent polygons whose areas are equal.
- An easy example of this is a triangle and a rectangle since the area of a triangle is one half the area of a rectangle with the same base and height.

Cone and Cylinder

Chicago Harbor Lighthouse-
Cylinder

Top of a house- Cone



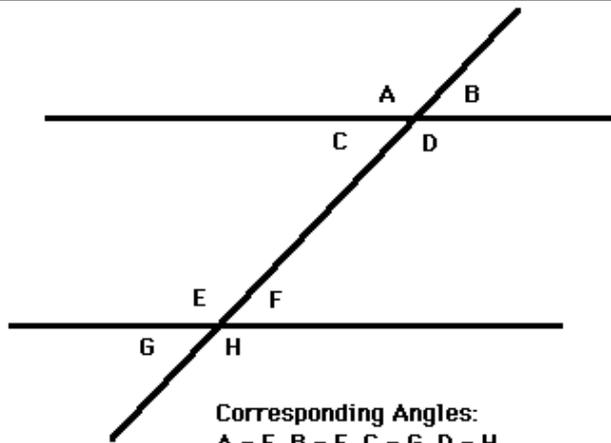
Concept

- If I shrink the image of the cone down to fit inside of the lighthouse it is much easier to visualize the relationship between the volumes.
- The volume of a cone is one-third the volume of a cylinder of the same base and height.

Analysis- Parallel Lines Cut by a Transversal



- If you rotate the diagram 90 degrees you can see the corresponding parallel and transversal lines.
- Using a protractor I found all the angles necessary to prove the diagram to the left. Labeling them the same way.
- $A = D = E = H = 140$ degrees
- $B = C = F = G = 40$ degrees



Corresponding Angles:
 $A = E, B = F, C = G, D = H$

Alternate Interior Angles:
 $C = F, D = E$

Alternate Exterior Angles:
 $A = H, B = G$

Analysis- Similar Triangles



- Using just a ruler, I measured the lengths of all sides of one of the smaller triangles.
- I also found the lengths of all sides of one of the larger triangles.
- The scale factor of these is 2.
- This was made easier by the fact that the smaller triangle was contained within the larger triangle and because they were right triangles, all the angles were also equal.

Analysis- Regular Polygon with more than 4 sides



- Using a protractor I found that one angle is approximately 108 degrees, taking into account some human error.
- Finding each angle individually I found that they are the same.
- The sum of all the angles is 540 degrees.
- This is equal to the equation $(n-2)180$, where $n=5$, which confirms the equation.

Geometry in Art

A regular polygon with more than four sides

Tri-hexa-flexagon,

Tetra-hexa-flexagon,

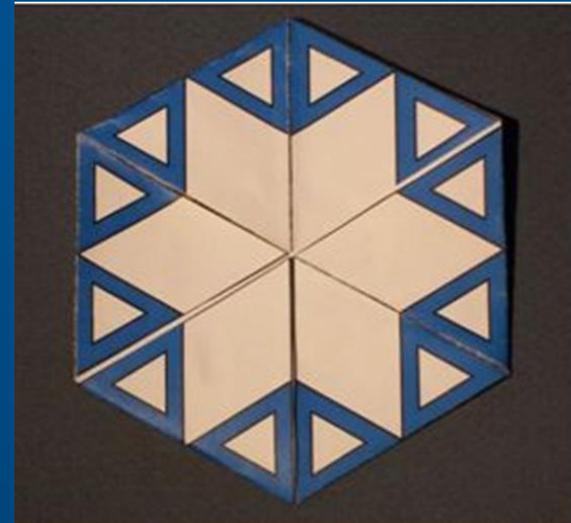
Hepta-hexa-flexagon,

or even

Penta-hexa-flexagon



Img Src: <http://isotropic.org/polyhedra/>

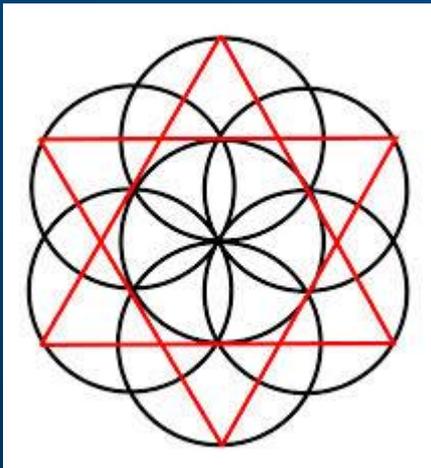


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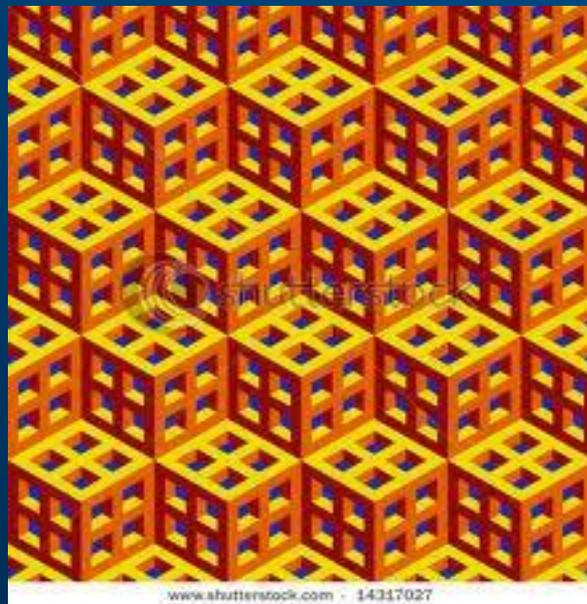
<http://britton.disted.camosun.bc.ca/trihexaflexagon/flexagon.html>

Polyhedra

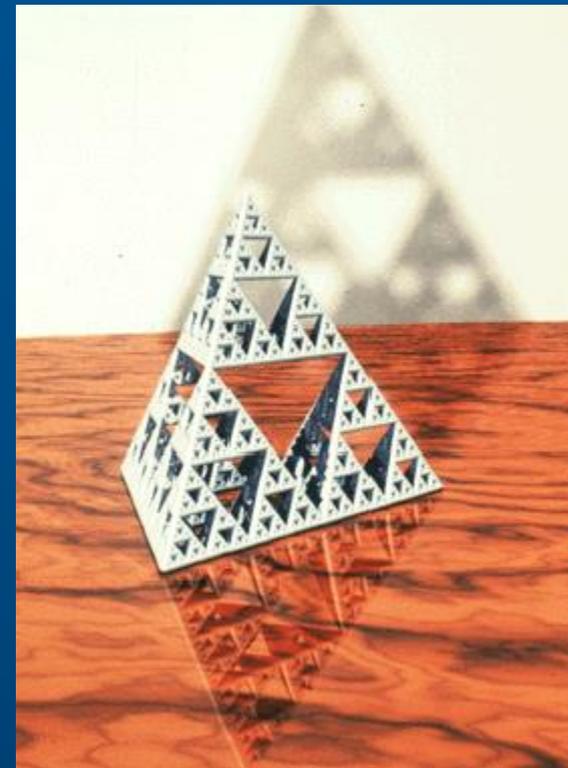
More Geometry in Art



Img Src:
www.soulofdistortion.nl

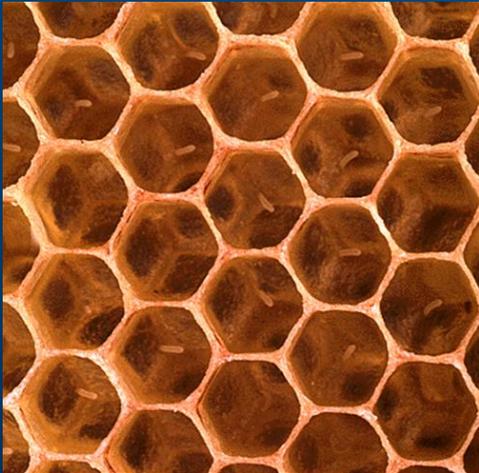


Img Src: tarantamath.pbworks.com



Img Src: www.dartmouth.edu

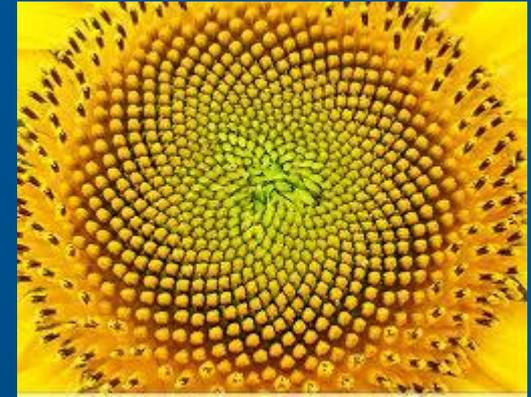
Geometry in Nature



Img Src: phandroid.com



Img Src: www.photographyireland.net



Img src:
www.spirituallifemagazine.com



Img Src: www.flickr.com



Img Src: photo.net

Conclusions and Summary

- This is a great real-world application of geometry and might give the students some relevancy to what they are doing.
- It is much easier to find the images in art and architecture than nature, which might be difficult depending on the grade level
- Possible issues would be with students who don't have digital cameras or access to the internet. Time would have to be given in class and in a computer lab to try and work with this situation. I would also try doing this in groups instead of individually.

Now what???

Small group Geometry scavenger hunt

- Get into groups of 2-3 people
- Each group will go with an IMSA Student
- Each group will receive a copy of the 10 geometry concepts to look for in the building
- Go on a scavenger hunt, taking your camera phones with you to take photos of the geometry concept
- The IMSA student is just to guide you, they will not take you directly to the place where you can find the geometry concept
- Most importantly- BE CREATIVE!

Adapting the lesson

- Get into groups based on grade level you teach
- Discuss how to adapt it to your own classroom- for example how could you do this with 3rd graders? How can I use this same idea in other math classes?
- Discuss the difficulties you see in doing this in your classroom- like not having access to computers
- Share with the entire group- if we brainstorm together we may be able to come up with creative solutions to the difficulties