## Parametric Worksheet

(To be done just before project is assigned.)

1. From its initial position at (3,4), an object moves linearly, reaching (9, 8) after two seconds and (15, 12) after four seconds.
a. Predict the position of the object after six seconds, nine seconds, and *t* seconds.

b. Is there a time when the object is equidistant from the coordinate axes? If so, when?

2. The parametric equations $x = -2 - 3t$ and $y = 6 + 4t$ describe the movement of a particle, in meters per second. How does the particle's position change each second? each minute? What is the speed of the particle in meters per second?

3. Point (x, y) described by the equations $x = 1 + 2t$ and $y = 3 + t$ form a line. Is the point (7, 6) on this line? How about (-3, 1)? How about (6, 5.5)? (11, 7)?

4. The x and y coordinates of a point are given by the following equations:

$$x = -4 + 3t$$
$$y = 1 + 2t$$

Sketch the points corresponding to *t* = -1, 0, and 2. These points should appear to be collinear. Convince yourself that this is the case, and calculate the slope of this line. The above equations are called *parametric*, and *t* is called the parameter. How is the slope of a line determined from its parametric equations?

5. Find parametric equations to describe the line that contains the points A = (5, -3) and B = (7, 1). There is more than one correct way to answer this question.

6. Find parametric equations that describe a line through (3, 1) and (7, 3).

7. Using your graphing calculator, graph the parametric equations

$$x = 22t$$
$$y = -4.9t^2 + 22t + 2$$

The above will give the path of a ball after *t* seconds, *x* and *y* being in meters.

a. Determine when the ball will hit the ground.

b. Determine how high the ball goes.

c. Will the ball clear a fence 100 meters away that is two meters high? Show the algebra that leads to your conclusion.

8. What is the difference between the following two sets of parametric equations, both of which graph into the same line.

$x = 5t - 2$ $\qquad$ $x = 10t - 2$

$y = -2t + 3$ $\qquad$ $y = -4t + 3$

Last term, in MI 3 you used parametric equations to model a Ferris wheel and a cycloid. Parametric equations are useful in modeling motion when different forces are at work in different directions. In this project you will be asked to model the flight of a ball. First some review of physics.

If an object is thrown vertically upward (ignoring wind resistance) its height can be modeled by the equation:

$h(t) = -16t^2 + v_0 t + h_0$ with $h$ in feet and $v$ in feet per second; or

$h(t) = -4.9t^2 + v_0 t + h_0$ with $h$ in meters and $v$ in meters per second

where
$t =$ seconds
$v_0 =$ initial velocity
$h_0 =$ initial height

$v(t)$ (the vertical velocity of the object) is given by
$v(t) = -32t + v_0$ in feet per second
$v(t) = -9.8t + v_0$ in meters per second

**Example:**

Aldo stands on the top of a building that is 50 ft tall and throws a ball upwards at 34 ft/sec towards Isolina who is on the ground. The height of the ball at time $t$ is given by

$h(t) = -16t^2 + 34t + 50$

The vertical velocity of the ball at time $t$ is given by

$v(t) = -32t + 34$

So at time $t = 2$ seconds, the ball is at 54 ft, with a velocity of -30 ft/sec (it is moving downward.)

Note: The implied domain is $\left[0, \frac{25}{8}\right]$.

The dimensions of Wrigley Field

Distances from plate:

Left field: 355 ft
Left-canter: 368 ft
Center field: 400 ft
Right-center: 368 ft
Right field: 353 ft
Plate screen: 60.5 ft

Scoreboard :

From plate (horizontal): 450 ft
Height: 27 ft
Width: 75 ft
Clock: 10 ft diameter
Its top is 85 feet above the field

No one has ever hit the scoreboard with a fly ball, although two people came close.

Trivial fact: When the last World Series game was played at Wrigley Field, only one of IMSA's math teachers was alive.

**1.** Adam Dunn comes to bat at Wrigley field. He hits the ball at a point three feet off the ground at an angle of $48°$ towards dead center field. The wall in center is 400 feet away, 11.5 high, and 1.5 foot thick. Note: for this problem, assume the Cubs play in a vacuum.

A. Suppose Dunn hits the ball at 114 feet/sec. Describe the flight of the ball in as thoroughly possible way as you can. (One example: how long is the ball in the air?) Include in your description the actual speed (not just vertical and horizontal) of the ball when it is no longer in flight, where it hits the ground, and other significant features of its flight. Explain the model you develop and show a graph of the flight of the ball.

B. At what speed must he hit the ball in order to just clear the wall?

**2.** (Adapted from Contemporary Precalculus) An air traffic controller is monitoring airplanes on a radar screen. The radar sweeps a circle every second, showing the location of planes flying in the vicinity. The screen is a 50 cm by 50 cm square. The controller notices that at a certain instant, one plane, A, is located along the left edge of the screen 40 cm from the bottom corner. Another plane, B, is located 45 cm to the right and 6 cm above the lower left corner. On the next rotation (1 second later), the controller observes the A is located at position 0.65 cm to the right and 0.30 cm below its previous position and B is 0.55 cm to the left and 0.50 cm above its previous position. Assume the planes continue to fly in the same direction at a constant speed while they are visible on the screen. Note: one centimeter on the radar corresponds to 800 feet in the air.

a) Use parametric equations to model the positions of each plane.

b) Give their speeds and headings.

c) Assuming the planes fly at the same altitude, will they collide? Justify your answer.

**Extensions**

Choose one of the following options to complete your project. You may do both options for extra credit.

I. Back to hitting the home run. Explore the relationship between the angle the ball leaves the bat, the speed it comes off the bat, and how far it travels 'on the fly'.

Some thoughts (you may ask your own questions as well as answer these prompts)
- What is the best angle (two decimal places) to hit the ball so it travels the farthest, assuming it is hit 3 ft off the ground and what is the distance it travels?
- At a fixed speed of 120 ft/sec, what range of angles would be possible for a ball to travel more than 400 feet (do not be concerned about the wall)?
-What angle and speed would the batter have to hit the ball in order to hit the scoreboard at Wrigley Field (which has never been done)? Note: pick a reasonable speed. If the ball were shot from a canon it would be fairly easy to hit the scoreboard.

II. Back to problem 2.

a) If the planes do not collide, how close do they come to each other?

b) At the time they are the closest to each other.

c) If the pilot of plane A looked out her window at plane B, what direction would she look?

Writing Assignment Guidelines

For this assignment you are to work with one other student to collaborate and develop your solutions. Each person is to have his/her own write up. We will follow the same guidelines we had for your MI 3 writing assignment which are repeated below:

For this problem, you are to write your solutions carefully and completely. The understanding is you will work with one other person. The primary purpose for this assignment is to emphasize your writing and your ability to explain your mathematical work clearly as if you were writing this problem to a colleague that knows as much mathematics as you.

Your paper should include the following:
A good drawing(s) with labels.
An explanation of your set-up and the work done.
The actual mathematics involved, paying particular attention to justifying the mathematics involved.
Generalizations supported by you work, with good justifications written in complete sentences.
Writing style will be a factor in your evaluation.

In addition you are you are expected to present a high quality looking document, i.e., all equations and expressions are produced using printed (not written) format. Any diagram should have a minimum of hand notation.

Since this is your second writing assignment this term, it will be averaged into your test/quiz average and not problem sets and be worth a typical test.

**Assignment:** You are to do problems 1, 2, and one of the extensions. You may do both extensions for extra credit. Indicate on your paper with whom you worked. You must submit both a hard copy and an electronic copy to Turnitin.com.

**Due Dates:** Friday, April 19 Draft of problem 1.
Friday, April 16 at the beginning of class.

**Collaboration:** It is expected you will work with your partner to develop your mathematics. If you need additional assistance, you may see Mr. Kammrath, Dr. Fogel, or Mr. Hurlbut. Under no circumstances are you to collaborate with another group, either asking for assistance or giving it, including letting another group read your work before the project is due.